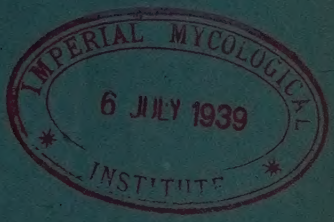
COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1939



VOLUME XXII

№ 9

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
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NOUVELLE SÉRIE

7-me année

Paraissant tous les dix jours

1939

VOLUME XXII, № 9

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Rédacteur en chef **A. Fersman.** Rédacteur **S. Bernstein.**

En composition 23/III 1939. En impression 8/V 1939. Nombre de feuilles d'impression 7¹/₄. Nombre de caractères par feuille d'impression 65 000. Pages 537—656. Format du papier 72×110 cm. Tirage 1 000. Glavlit A 771. Numéro d'édition 2067. Numéro typographique 420.

Imprimerie № 16. Moscou. Trekhproudnij péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MATHEMATICS

ON THE FOURIER SERIES OF GENERALIZED ALMOST PERIODIC FUNCTIONS

By B. LEWIN and B. LEWITAN

(Communicated by S. N. Bernstein, Member of the Academy, 27. I. 1939)

1. It is known that the almost periodic functions of H. Bohr — as well as all generalizations of them proposed hitherto — admit a mean value given by

$$M\{f\} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(x)\,dx.$$

One of us considered recently a new class of continuous almost periodic functions, in the domain of which the uniqueness theorem for Fourier series remains valid (1).

The generalized (in our sense) almost periodicity of a fonction $f(x)$ is defined as follows:

For any $\varepsilon > 0$ *and* $N > 0$ *there exists an almost periodic set of numbers* (2) $\tau(\varepsilon, N)$ *such that*

$$|f(x-\tau) - f(x)| < \varepsilon \qquad (|x| < N). \tag{1}$$

The functions satisfying this condition we shall call N-almost periodic functions.*

The numbers τ we schall call (ε, N)—translations.

The main results obtained in the paper referred to above are as follows:

a) The class of N-almost periodic function is linear.

---

* In the paper referred to under (1) is given a different definition of the generalized almost periodicity, the class of functions considered there being subjected to the following two conditions:

Condition I. Corresponding to any $\varepsilon > 0$ and $N > 0$ there exists a relatively dense set of numbers $\tau = \tau(\varepsilon, N)$ such that

$$|f(x \pm \tau) - f(x)| < \varepsilon \qquad (|x| < N). \tag{2}$$

Condition II. If for τ' resp. τ'' (2) holds with ε' resp. ε'', then for $\tau = \tau' + \tau''$ we have

$$|f(x \pm \tau) - f(x)| < \delta \qquad (|x| < N),$$

where $\delta = \delta(\varepsilon', \varepsilon'')$ tends to zero with ε' and ε''.

This class of functions coincides, however, under some sufficiently general assumptions with that of the N-almost periodic functions.

b) If an N-almost periodic function $f(x)$ satisfies the inequality

$$\overline{\lim_{T\to\infty}} \frac{1}{2T} \int_{-T}^{T} |f(x)|^2 \, dx < \infty \tag{3}$$

and if the limit

$$a(\lambda) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(x) e^{-i\lambda x} dx \tag{4}$$

exists for all real λ, then to $f(x)$ there corresponds a Fourier series

$$f(x) \sim \sum A_n e^{i\lambda_n x}$$

where

$$A_n = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(x) e^{-i\lambda_n x} \, dx,$$

and the function $f(x)$ is uniquely determined by its Fourier series.

If for a function $f(x)$ the limit (4) exists not for all values of λ, then the mean value adopted in Bohr's theory may be replaced by any functional $M\{f\}$ satisfying the following conditions:

1) $M\{af+bg\} = aM\{f\} + bM\{g\}$ [where a, b are constants and f, g satisfy (3)];

2) $M\{f\} \geqslant 0$, if $f \geqslant 0$;

3) $M\{f(x+x_0)\} = M\{f(x)\}$ (x_0 being real);

4) if $\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(x)\,dx$ exists, this limit coincides with $M\{f\}$.

In the most general case such a functional may be constructed by means of the generalized limit of S. Banach [3].

The functional $M\{f\}$ associates with any function subjected to (3) a Fourier series

$$f(x) \sim \sum A_n e^{i\lambda_n x}$$

where

$$A_n = M\{f(x) e^{-i\lambda_n x}\}.$$

c) If the Fourier series of two N-almost periodic functions constructed with the help of the same functional are identical, the functions are identical too.

d) For the functions in question holds a theorem of approximation by polynomials constructed after Weyl with the help of their Fourier series.

Now the natural question arises whether there exists a bounded N-almost periodic function, for which the ordinary mean value does not exist. In the present note we schall construct an example of such a function.

It is also to be noticed that the function constructed here is even an N-periodic function, i. e. to any number $N > 0$ there corresponds a number l_N such that

$$f(x + nl_{1N}) = f(x) \qquad (|x| < N, \quad n = 0, \pm 1, \pm 2, \ldots).$$

2. Let $\{\varepsilon_i\}$ be a sequence of positive numbers such that

$$\sum_{i=1}^{\infty} \varepsilon_i = \varepsilon < \frac{1}{3}.$$

Let us fix an arbitrary number $T_1 > 1$. Then we put $f(x) = 1$ on the interval $(-T_1, T_1)$.

Choosing now such a number l_1 that $l_1 > \frac{T_1}{\varepsilon_1}$, we put

$$f(x + nl_1) = f(x) \qquad (|x| < T_1, \quad n = 0, \pm 1, \pm 2, \ldots).$$

Thus, the function $f(x)$ is now defined on a periodic set of intervals $\mathfrak{M}_1$ with the density $< \varepsilon_1$ *.

Let now $T_2 > 0$ be a number such that the sum of lengths of the intervals, on which $f(x)$ is already defined and which fall into the interval $(-T_2, T_2)$, is $\leqslant 2\varepsilon_1 T_2$. The number T_2 must exist, since the density of the set does not exceed ε_1.

On the intersection of the set complementary to $\mathfrak{M}_1$ with the interval $(-T_2, T_2)$ we put $f(x) = 0$.

Now we choose a number $l_2 = n_1 l_1$ (n_1 being a positive integer) such that

$$l_2 > \frac{T_2}{\varepsilon_2}$$

and put

$$f(x + nl_2) = f(x) \quad \text{for } |x| < T_2, \quad n = 0, \pm 1, \pm 2, \ldots$$

Observe that the function $f(x)$ will not be overdetermined on the set $\mathfrak{M}_1$, since $l_2 = n_1 l_1$ is a period of the set $\mathfrak{M}_1$. The function $f(x)$ is now defined on a periodic set of interval $\mathfrak{M}_2$ with the density $< \varepsilon_1 + \varepsilon_2$.

Let now $T_3 > 0$ be a number such that sum of lengths of the intervals, on which the function $f(x)$ is already defined and which fall into the interval $(-T_3, T_3)$, is $\leqslant 2(\varepsilon_1 + \varepsilon_2) T_3$.

We choose then a number $l_3 = n_2 l_2$ (n_2 being a positive integer) satisfying the condition

$$l_3 > \frac{T_3}{\varepsilon_3}.$$

On the intersection of the set complementary to $\mathfrak{M}_2$ with the interval $(-T_3, T_3)$ we put $f(x) = 1$ and

$$f(x + nl_3) = f(x) \quad \text{for } |x| < T_3, \quad n = 0, \pm 1, \pm 2, \ldots$$

Observe that the function $f(x)$ will not be overdetermined on the set $\mathfrak{M}_2$. The function $f(x)$ is thus defined on a periodic set of intervals $\mathfrak{M}_3$ with the density $< \varepsilon_1 + \varepsilon_2 + \varepsilon_3$.

Thus, we are able to determine in succession

$$T_1, T_2, T_3 \ldots \text{ and } l_1, l_2, l_3, \ldots$$

If we proceed in this way indefinitely, we shall obtain a function $f(x)$ defined and bounded on the whole axis.

---

* As usual, we mean by the density of a set E the $\lim_{t \to \infty} \frac{\text{mes } E_t}{2t}$, where E_t denotes the intersection of E and $(-t, t)$.

We are going to show that the function $f(x)$ does not admit a mean value. In fact, we have

$$\frac{1}{2T_1}\int_{-T_1}^{T_1} f(x)\,dx = 1 > \frac{2}{3},$$

$$\frac{1}{2T_2}\int_{-T_2}^{T_2} f(x)\,dx \leqslant \varepsilon_1 < \frac{1}{3},$$

$$\frac{1}{2T_3}\int_{-T_3}^{T_3} f(x)\,dx \geqslant 1-(\varepsilon_1+\varepsilon_2) > 1-\varepsilon > \frac{2}{3}.$$

and, in general,

$$\frac{1}{2T_{2k+1}}\int_{-T_{2k+1}}^{T_{2k+1}} f(x)\,dx \geqslant 1-(\varepsilon_1+\varepsilon_2+\ \dots\ +\varepsilon_{2k}) > 1-\varepsilon > \frac{2}{3},$$

$$\frac{1}{2T_{2k}}\int_{-T_{2k}}^{T_{2k}} f(x)\,dx \leqslant \varepsilon_1+\varepsilon_2+\ \dots\ +\varepsilon_{2k-1} < \varepsilon < \frac{1}{3}.$$

Consider now the function

$$f_\delta(x) = \frac{1}{\delta}\int_0^\delta f(x+u)\,du \qquad (0<\delta<1).$$

Obviously $f_\delta(x)$ is a bounded uniformly continuous function. We assert that the mean value does not exist for $f_\delta(x)$. In fact, this follows from the identity

$$\frac{1}{2T}\int_{-T}^{T} f_\delta(x)dx = \frac{1}{2T}\int_{-T}^{T} dx\,\frac{1}{\delta}\int_0^\delta f(x+u)\,du = \frac{1}{\delta}\int_0^\delta du\,\frac{1}{2T}\int_{-T}^{T} f(x+u)\,dx.$$

Let us finally show the N-periodicity of the function $f_\delta(x)$. Since any interval $(-N, N)$ lies within some interval $(-T_m, T_m)$ in which

$$f(x+nl_m) = f(x) \quad (|x|<T_m, \quad n=0, \pm 1, \pm 2, \dots),$$

we have

$$f_\delta(x+nl_m) = f_\delta(x) \quad (|x|<T_m-\delta, \quad n=0, \pm 1, \pm 2, \dots),$$

and the property in question is established.

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Received
1. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MATHEMATICS

ON THE k-LINEAR OPERATIONS IN BANACH SPACES

By M. GOVURIN

(Communicated by S. N. Bernstein, Member of the Academy, 31. I. 1939)

The object of the present paper is the investigation of k-fold linear operations in Banach spaces and the construction of a kind of algebra of these operations, which will be found useful in the theory of differential and integral calculus in Banach spaces (2).

I. Let $X_1, X_2, \dots, X_k \cdot Z$ be $k+1$ Banach spaces. An operation $z = u_k(x_1, x_2, \dots, x_k)$ defined for $x_i \in X_i$ $(i = 1, 2, \dots, k)$ and with values belonging to Z, is called k-fold linear or k-linear, if it is: 1) additive with respect to each of its arguments and 2) bounded, i. e. a number M exists such that (whatever be $x_1 \in X_1$, $x_2 \in X_2, \dots, x_k \in X_k$)

$$\| u_k(x_1, x_2, \dots, x_k) \| \leqslant M \| x_1 \| \cdot \| x_2 \| \dots \| x_k \|. \tag{1}$$

From these two conditions follows the continuity of a k-linear operation u_k with respect to all of its arguments simultaneously and the identity

$$\begin{aligned} u_k(x_1, x_2, \dots, x_{i-1}, ax_i' + bx_i'', x_{i+1}, \dots, x_k) = \\ = au_k(x_1, x_2, \dots, x_{i-1}, x_i', x_{i+1}, \dots, x_k) + \\ + bu_k(x_1, x_2, \dots, x_{i-1}, x_i'', x_{i+1}, \dots, x_k). \end{aligned} \tag{2}$$

The least M satisfying the condition (1) (for all $x_1, x_2, \dots, x_k$) is called the norm of the operation u_k and denoted by $\| u_k \|$. Evidently

$$\| u_k \| = \sup_{x_i \in X_i, \| x_i \| = 1} \| u_k(x_1, x_2, \dots, x_k) \|. \tag{3}$$

The set of all k-linear operations with arguments belonging to $X_1, \dots, X_k$ and values belonging to Z may be transformed into a Banach space $\mathfrak{U}_k = \{X_1, \dots, X_k \to Z\}$ with the evident definition of the sum of two elements, the product of an element into a real number and with the norm defined by (3).

In the sequel we shall suppose that any two spaces X_i, X_j either coincide or possess no elements in common.

If an operation $u_k \in \mathfrak{U}_k$ does not change its value when we transpose two of its arguments belonging to any two coinciding spaces X_i, X_j, then we shall say that u_k is a symmetrical operation. From this definition it follows that if all the spaces $X_1, \dots, X_k$ are different, then every operation $u_k \in \mathfrak{U}_k$ is symmetrical. We shall denote the space U_k of

all symmetrical operations belonging to $\mathfrak{U}_k$ by $[X_1, \dots, X_k \to Z]$. It is a closed linear subspace of $\mathfrak{U}_k$. The existence of symmetrical k-linear operations different from the zero operation can be easily proved for any given spaces $X_1 \dots X_k$ and Z.

II. If in the operation $Z = u_k(x_1, \dots, x_k)$ we fix $l < k$ arguments, supposing, e. g., that $x_1 = \bar{x}_1, \dots, x_l = \bar{x}_l$, we shall obtain a $k-l$-linear operation, viz. $u_k(\bar{x}_1, \dots, \bar{x}_l, x_{l+1}, \dots, x_k)$ with arguments $x_{l+1}, \dots, x_k$, i. e. an element of the space $\{X_{l+1}, \dots, X_k \to Z\}$. Denoting this operation by v_{k-l}, we see that the operation u_k generates a new operation

$$v_{k-l} = u_l^*(\bar{x}_1, \dots, \bar{x}_l),$$
$$u_l^* \in \{X_1, \dots, X_l \to \{X_{l+1}, \dots, X_k \to Z\}\},$$

and we have $\| u_k \| = \| u_l^* \|$.

If the operation u_k is symmetrical, then the operations u_l^* and v_{k-l} are also symmetrical.

III. The following definition will be found very convenient. Let w be an arbitrary (fixed) element of the space $[X_1, \dots, X_k \to Z]$ such that $\| w \| = 1$. The element $z = w(x_1, \dots x_k)$ we shall call the product of the elements $x_1, \dots, x_k$ and shall denote it by

$$x_1 x_2 \dots x_k. \tag{4}$$

The operation w being symmetrical, we can suppose this product to be commutative.

The choice of the element w determines the type of the multiplication. We shall consider more closely the following two types.

IV. Products of the First Type. Let $X_1, \dots, X_k$ be arbitrary Banach spaces. Consider the set Ξ^*, the elements of which are linear combinations

$$\xi = \sum_{i=1}^{p} a_i x_1^{(i)} \dots x_k^{(i)} \quad p = 1, 2, \dots, x_s^{(i)} \in X_s \tag{5}$$

where a_i are arbitrary real numbers and $x_j^{(i)}$ — arbitrary elements $\in X_j$.

The definitions of the sum of two elements of Ξ^* and of the product of an element Ξ^* into a real number are evident.

The norm of the element $\xi \in \Xi^*$ we shall define as

$$\| \xi \| = \| \sum_{i=1}^{p} a_i x_1^{(i)} \dots x_k^{(i)} \| = \sup_{\| f \| = 1} \left| \sum_{i=1}^{p} a_i f(x_1^{(i)}, \dots, x_k^{(i)}) \right|, \tag{6}$$

where sup is taken for all symmetrical k-linear functional f with the norm equal to 1. It is easy to prove that if Z is an arbitrary Banach space having at least one element different from 0, then

$$\| \xi \| = \sup_{u_k \in [X_1, \dots, X_k \to Z],\ \| u_k \| = 1} \| \sum_{i=1}^{p} a_i u_k(x_1^{(i)}, \dots x_k^{(i)}) \|.$$

Of course, Ξ^* is not as yet a linear space; but if we identify in Ξ^* the elements ξ and ξ' such that $\| \xi - \xi' \| = 0$, we shall obtain a linear normed space though generally not complete. If we complete it in the ordinary way, i. e. by adjoining as non-proper elements of the sets of all equivalent fundamental sequences of elements of Ξ^*, we shall obtain a

new space Ξ, which is linear, normed and complete, i. e. a Banach space.

Put now $Z=\Xi$ and consider the k-linear operation $w \in [X_1, \dots, X_k \to \Xi]$ defined as follows:

$$w(x_1, \dots, x_k) = x_1 \dots x_k.$$

It is evident that $\|w\|=1$ and that w is a symmetrical operation. If Z and w are defined as above, then the product ot the elements $x_1 x_2 \dots x_k$ is an element $x_1 x_2 \dots x_k \in \Xi$.

V. Products of the Second Type. Let $X_1, \dots X_{k-1}$ and Z be k arbitrary Banach spaces and put $X_k = [X_1, \dots, X_{k-1} \to Z]$; define now the operation $w \in [X_1, \dots, X_{k-1}, X_k \to Z]$ as follows:

$$w(x_1, \dots, x_{k-1}, x_k) = x_k(x_1, \dots, x_{k-1})$$

for $x_i \in X_i, \quad i=1, 2, \dots, k$.

The operation x_k being symmetrical, the same is true for the operation w; besides, we have evidently $\|w\|=1$. Thus, the expression $u_{k-1}(x_1, \dots, x_{k-1})$, where $u_{k-1} \in [X_1, \dots, X_{k-1} \to Z]$ can be written as the product $u_{k-1} \cdot x_1 \dots x_{k-1}$.

Observe that this type of products is in a certain sense the most general type, because in any product $x_1 \dots x_k$ we can consider any factor x_i (with a fixed index i) as the symbol of an operation effected over $k-1$ other factors and defined as

$$x_i(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_k) = x_1 \dots x_k.$$

In what follows we shall always write the k-linear operations in the multiplicative form, i. e. we shall write $u_k(x_1, \dots x_k)$ instead of $u_k x_1 \dots x_k$. Also (unless the reverse is expressely stated or follows from the context) we shall mean the product in the sense of IV, when no operations occur among its factors. In the case when one of the factors is an operation, the product will be understood in the sense of V.

We shall now introduce some new conditions which are necessary for the construction of an algebra of these products.

VI. We shall call the space Ξ introduced in IV the product of the spaces $X_1, \dots, X_k$ and denote it by

$$X_1 \dots X_k.$$

To every symmetrical k-linear operation $u_k \in (X_1, \dots, X_k \to Z)$ corresponds a linear operation $u \in \{\Xi \to Z\} = [\Xi \to Z]$ defined as follows: if ξ is proper element of Ξ and (5) is its representation, then

$$u\xi = \sum_{i=1}^{p} a_i u_k x_1^{(i)} \dots x_k^{(i)}$$

(by this condition the linear operation is completely determined for proper, as well as for non-proper elements of Ξ).

Conversely, any operation $u \in \{\Xi \to Z\}$ corresponds to the k-linear symmetrical operation $u_k \in [X_1, \dots, X_k \to Z]$ defined by the expression

$$u_k x_1 \dots x_k = u \cdot (x_1 \dots x_k),$$

where $x_1 \dots x_k$ on the right-hand side belong to Ξ. Since, moreover, $\|u_k\| = \|u\|$, it follows that the spaces $[X_1, \dots, X_k \to Z]$ and $\{X_i \dots X_k \to Z\}$ are isomorphic and we may consider them as coinciding.

The expression $u_k x_1 \dots x_l$ $(l < k)$ as yet has no sense, but if we want to give it a sense in such a way that the expression $(u_k x_1 \dots x_l)\, x_{l+1} \dots x_k$ coincides with $u_k x_1 \dots x_l x_{l+1} \dots x_k$, then we must suppose, according to V, that $u_k x_1 \dots x_l \in [X_{l+1}, \dots, X_k \to Z] = \{X_{l+1} \dots X_k \to Z\}$ and, in particular, that the value of $u_k x_1 \dots x_l$ is the $(k-l)$-linear operation $v_{k-l} \in [X_{l+1}, \dots X_k \to Z]$ defined by the formula $v_{k-l} x_{l+1} \dots x_k = u_k x_1 \dots x_l x_{l+1} \dots x_k$ (cf. II).

VII. The product $(x_1 \dots x_l)\,(x_{l+1} \dots x_k)$ belongs, by V, to the space $(X_1 \dots X_l)\,(X_{l+1} \dots X_k)$, but since this space can be considered as coinciding with the space $X_1 \dots X_l X_{l+1} \dots X_k$, we may suppose that the product $(x_1 \dots x_l)\,(x_{l+1} \dots x_k)$ coincides with the element $x_1 \dots x_l x_{l+1} \dots x_k$ of the space $X_1 \dots X_l \cdot X_{l+1} \dots X_k$.

VIII. In the case when all the spaces $X_1, \dots, X_k$ coincide, we shall write X^k instead of $\underbrace{X \dots X}_{k}$ and x^k instead of $\underbrace{x \dots x}_{k}$.

We can say that with the help of the first and the second types of product an algebra may be constructed, in which all ordinary properties of addition and multiplication hold. The following conditions must then be satisfied: a) the sum will be considered only when all its terms are uniform, i. e. when they all belong to the same space; b) the consecutive product of several factors will be considered only when the product of all these factors taken simultaneously exists [for example, if $u \in \{X \to Z\}$, $x_1, x_2 \in X$, we cannot consider the product $x_1\,(u x_2) \in XZ$, since $x_1 \cdot u \cdot x_2$ is not defined].

We remind the reader that it is necessary that all operations considered are symmetrical. Take, e. g., Newton's binomial

$$(x_1 + x_2)^n = \sum_{=0}^{n} \binom{n}{k} x_1^{n-k} \cdot x_2^k \tag{7}$$

Both members of this identity can be multiplied by an arbitrary symmetrical n-linear operation u_n, because the left-hand side and any summand of the right-hand side is an element of the space $\Xi = X^n$ and u_n may be considered as a linear operation in Ξ. However, if u_n is not a symmetrical operation, then the identity (7) cannot be multiplied by u_n.

IX. In conclusion we propose the following problem. Let $u_k \in \{X^k \to Z\}$ and put

$$\| u_k \|_s = \sup_{\| x \| = 1} \| u_k x^k \|.$$

Then $\| u_k \|_s \leqslant \| u_k \|$ [cf. (2)]. A simple example shows that (in some cases) we can have $\| u_k \|_s < \| u_k \|$. Let $X = M$ be the space of bounded measurable functions $x(t)$ $(0 \leqslant t \leqslant 1)$ with the norm defined as

$$\| x \| = \operatorname{vrai} \max_{0 \leqslant t \leqslant 1} | x(t) |,$$

Z be the space of real numbers and

$$u_2 x_1 x_2 = u_2 x_2 x_1 = \int_0^{\frac{1}{2}} x_1(t)\, x_2(t)\, dt - \int_{\frac{1}{2}}^{0} x_1(t)\, x_2(t)\, dt.$$

It may be easily seen that $\| u_2 \| = 1$, while $\| u_2 \|_s = \frac{1}{2}$.

It would be interesting to find some lower bound for the ratio $\frac{\|u_k\|_s}{\|u_k\|}$. Banach * showed (1) that in the case when $X=Z=L^2$, we always have $\|u_k\|_s=\|u_k\|$. We can show that for the imaginary-linear spaces (i. e. such spaces, in which the elements may be multiplied by imaginary quantities) the following inequality holds:

$$\frac{\|u_k\|_s}{\|u_k\|} \geqslant \frac{k!}{k^k}. \tag{8}$$

This inequality follows from the identity

$$x_1^{k-1}x = \frac{1}{k^2}\sum_{l=0}^{k-1} e^{-\frac{2\pi l i}{k}}\left(x_1 + xe^{\frac{2\pi l i}{k}}\right)^k,$$

which gives

$$x_1 \dots x_k = \frac{1}{(k!)^2}\sum_{l_2=0}^{1}\sum_{l_3=0}^{2}\cdots\sum_{l_k=0}^{k-1} e^{-2\pi i\left(\frac{l_2}{2}+\frac{l_3}{3}+\dots+\frac{l_k}{k}\right)} \times$$

$$\times\left(x_1 + x_2 e^{\frac{2\pi l_1 i}{2}} + \dots + x_k e^{\frac{2\pi l_k i}{k}}\right)^k. \tag{9}$$

If we now put $\|x_1\| = \dots = \|x_k\| = 1$, every summand in (9) has a norm not exceeding k. Multiply both sides of (9) by u_k. Then every summand on the right-hand side will have a norm not exceeding $\|u_k\|_s k^k$, while their number is $k!$. Consequently, the left-hand side of (9) has a norm not exceeding $\frac{k^k}{k!}\|u_k\|_s$. Hence follows the required inequality

$$\|u_k\| \leqslant \frac{k^k}{k!}\|u_k\|_s.$$

In particular

$$\frac{\|u_2\|_s}{\|u_2\|} \geqslant \frac{1}{2}, \quad \frac{\|u_3\|_s}{\|u_3\|} \geqslant \frac{2}{9}.$$

These last inequalities hold for real linear spaces. In the case $k=2$ the coefficients ot the right-hand side of (9) are real, and for $k=3$ we may apply the following identity:

$$x_1x_2x_3 = \frac{1}{24}\left[(x_1+x_2+x_3)^3 + (-x_1+x_2+x_3)^3 + \right.$$
$$\left. + (x_1-x_2+x_3)^3 + (x_1+x_2-x_3)^3\right].$$

The example at the beginning of this § show that for $k=2$ the above lower bound $\left(\frac{1}{2}\right)$ is exact. It would be interesting to find whether for $k>2$ the lower bound given by (8) is exact; it would be also interesting to find any lower bound for real linear spaces (for $k>3$). On the other hand, we could (following Banach's example) investigate different particular classes of linear spaces.

Received
8. II. 1939.

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* Banach gives a different definition of a symmetrical operation and proves the identity $\|u_k\|_s = \|u_k\|$ for operations symmetrical in his sense. Nevertheless, his result may be applied also to operations symmetrical in our sense.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MATHEMATICS

ON THE DIFFERENTIAL AND INTEGRAL CALCULUS IN BANACH SPACES

By M. GOVURIN

(Communicated by S. N. Bernstein, Member of the Academy, 31. I. 1939)

I. In this paper X will denote a Banach space. Under a curve L in X we understand the set of all continuous functions of bounded variation $x(t)$ [where t is a real variable, $a \leqslant t \leqslant b$, and, for any t, $x(t) \in X$] which can be transformed into each other by substitutions of the form $t = \theta(\tau)$, $\alpha \leqslant \tau \leqslant \beta$, where $\theta(\tau)$ is a continuous monotonous function [as to the definition of «a function of bounded variation», see (2)]. Every such function $x(t)$ we shall call a representation of the curve L. The variation $\operatorname{var} x(t)$ of $x(t)$ in the interval $(a; b)$ (which is the same for all representations of the curve L) we shall call the length of L and denote by $|L|$.

The meaning of such expressions as the «initial» and the «end point» of a curve, a «closed» curve, a «segment», a «broken line», a broken line «inscribed into» a curve, a point «lies on» a curve, is evident. We shall say that a sequence $\{L_n\}$ of curves has a curve L for its limit, if there exists such a sequence $x_n(t)$ of representations of these curves and such a representation $x(t)$ of L that all $x_n(t)$ and all $x(t)$ are defined in the same interval $(a; b)$ and $x_n(t) \rightarrow x(t)$ uniformly. It is almost evident that for every curve L we can find a sequence $\{L_n\}$ of broken lines inscribed into L and having L for its limit. It is also easy to see that the length of any broken line inscribed into a curve L cannot exceed the length $|L|$ of the curve itself.

II. In a previous note (2) we have defined the meaning of the expressions:

$$\int_a^b x(t)\, du(t) \quad \text{and} \quad \int_a^b u(t)\, dx(t), \tag{1}$$

where $x(t)$, $a \leqslant t \leqslant b$, is a continuous function with values belonging to X, and $u(t)$ is a function of bounded variation with values belonging to the space $U = \{X \rightarrow Z\}$ (3), where Z is also a Banach space. The values of both integrals belong to Z.

In the sequel we shall suppose that the reader is acquainted with both our notes (2, 3) and, in particular, with the definition and properties of both integrals (1). We shall only remark here that the first of these integrals does not exceed $\max \|x(t)\| \operatorname{var} u(t)$. It is important to note that the properties of this integral remain the same in whatever way we define the product ux, because any product can be considered as a product of the second type [(3), V].

Let $\varphi(x)$ be an arbitrary continuous function defined in a certain sphere $S \subset X$ with values belonging to $U = \{X \to Z\}$ *. If $x(t)$ $(a \leqslant t \leqslant b)$ is a representation of a curve L in X and if $L \subset S$, then $\varphi(x(t))$ is a continuous function, and the integral

$$\int_a^b \varphi(x(t))\, dx(t) \tag{2}$$

exists.

The value of this integral depends only on the curve L and not on its special representation $x(t)$; we shall call this integral the curvilinear integral of the function $\varphi(x)$ along the curve L and denote it by

$$\int_L \varphi(x)\ dx. \tag{3}$$

III. In the case when the integral (3) has the same value for any two curves L and L_1, both lying in S and having the same initial point x_1 and the same end point x_2, we shall say that the integral (3) does not depend on the path of integration and denote this integral as follows:

$$\int_{x_1}^{x_2} \varphi(x)\ dx. \tag{4}$$

In order that the integral (3) should be independent of the path, it is evidently necessary and sufficient that it should vanish on any triangle lying in S. Let $Z = X^k$, $U = X^{k-1}$, $S = X$, $\varphi(x) = x^{k-1}$. It may be easily seen that the integral

$$\int_L x^{k-1}\ dx \tag{5}$$

vanishes, if L is a triangle. Consequently, this integral does not depend on the path and we have

$$\int_{x_1}^{x_2} x^{k-1}\ dx = \frac{1}{k}(x_2^k - x_1^k).$$

If $u_k \in [X^k \to Z]$, then we have

$$u_k \int_L x^{k-1}\ dx = \int_L u_k\ x^{k-1}\ dx,$$

since the integral is the limit of certain sums (2) and any symmetrical operation is distributive with respect to the addition (3). Therefore the integral $\int u_k x^{k-1} dx$ is independent of the path provided that u_k is a symmetrical operation.

When $k = 2$, the converse is also true, i. e. if the integral

$$\int u_2 x\ dx$$

does not depend on the path of integration, then the operation u_2 is symmetrical.

In fact, simple calculation gives us the value of the integral $\int u_2 x\, dx$ along a parallelogram with vertices at 0, $x_1, x_1 + x_2$ and x_2 as equal to

* Instead of supposing φ to be defined on a sphere, we could suppose it to be defined on a set $M \subset X$ satisfying certain condition, but we shall not enter into this here.

$u_2x_1x_2 - u_2x_2x_1$. If this difference is 0 for any x_1, x_2, then u_2 is a symmetric operation.

IV. The differential of an abstract function has already been defined (by Fréchet). It will be found useful to consider (along with the differentials) the derivatives of abstract functions. We shall say that a linear operation $u \in \{X \to Z\}$ is the derivative of a function $z = \varphi(x)$ at the point, if the linear (with respect to Δx) expression $u\Delta x$ is the principal part of $\Delta\varphi(x) = \varphi(x+\Delta x) - \varphi(x)$, i. e. if

$$\varphi(x+\Delta x) - \varphi(x) = u\Delta x + \rho(\Delta x),$$

where $\frac{\|\rho(\Delta x)\|}{\|\Delta x\|} \to 0$ as $\|\Delta x\| \to 0$. This derivative we shall denote in the usual way, i. e. we shall write

$$u = \frac{d\varphi(x)}{dx} = \varphi'(x).$$

Suppose that $\varphi(x)$ possesses the derivative $\varphi'(x)$ at every point of S and that the second derivative $\varphi''(x) = (\varphi'(x))'$ exists. Then the values of $\varphi''(x)$ lie in $\{X \to \{X \to Z\}\}$, i. e. in $\{X, X \to Z\}$. Generally, if we set $\varphi^{(n)}(x) = (\varphi^{(n-1)}(x))'$, then $\varphi^{(n)}(x) \in \{\underbrace{X, \ldots, X}_{n} \to Z\}$. In particular, by a direct computation we find that if $Z = X^k$, then $(x^k)^{(n)} = k(k-1)\ldots(k-n+1)x^{k-n}$; hence, when $n > k$, $(x^k)^{(n)} = 0$.

Many of the properties of derivatives in the ordinary differential calculus remain true for these generalized derivatives. Thus, e. g., if $\varphi(x) = \psi(\theta(x))$ $(\theta(x) \in X_1)$, then $\varphi'(x) = \psi'(\theta(x))\,\theta'(x)$, where, of course, $\theta' \in \{X \to X_1\}$ and $\psi' \in \{X \to Z\}$, while the product $\psi'\theta'$ must be taken as the ordinary — non commutative — product of operations.

V. Let us calculate the integral

$$\int_L \varphi'(x)\,dx, \tag{6}$$

where L is the segment $[x_1; x_2]$; one of the representations of L is $x(t) = x_1 + t(x_2 - x_1)$, $0 \leqslant t \leqslant 1$, so that we have

$$\int_L \varphi'(x)\,dx = \int_0^1 \varphi'(x_1 + t(x_2 - x_1))(x_2 - x_1)\,dt = \int_0^1 \psi'(t)\,dt = \psi(1) - \psi(0) =$$

$$= \varphi(x_2) - \varphi(x_1),$$

where $\psi(t) = \varphi(x_1 + t(x_2 - x_1))$ and the value of the integral $\int \psi'(t)\,dt$ is furnished by Bochner's theorem (1). If L is a broken line and x_1 and x_2 are its end points, then, as we immediately see, the value of the integral (6) remains $\varphi(x_2) - \varphi(x_1)$. Consequently, the integral (6) does not depend on the way of integration and its value is always the difference of the values of $\varphi(x)$ at the end points of L.

Theorem. *If in a neighbourhood S of a point x_0 (including this point itself) there exists the second derivative $\varphi''(x)$ of $\varphi(x)$, then $\varphi''(x_0) = u_2$ is a symmetrical operation.*

Proof. Denote by L the contour of the parallelogram having the points $x_0, x_0 + x_1, x_0 + x_1 + x_2$ and $x_0 + x_2$ for its vertices. Then, if $L \subset S$, we have $\int_L \varphi'(x)\,dx = 0$. Suppose that for $\|x - x_0\| < a$

$$\|\varphi'(x) - \varphi'(x_0) - \varphi''(x_0)(x - x_0)\| = \|\rho(x - x_0)\| \leqslant \varepsilon\|x - x_0\|.$$

We have then

$$0=\int_L \varphi'(x)\,dx=\int_L \varphi'(x_0)\,dx+\int_L \varphi''(x_0)(x-x_0)\,d(x-x_0)+\int_L \rho(x-x_0)\,dx. \quad (7)$$

If we substitute λx_1, λx_2 $(\lambda>0)$ for x_1 and x_2 respectively and denote the corresponding contour by L_λ, then

$$\int_{L_\lambda} \varphi''(x_0)(x-x_0)\,d(x-x_0)=\lambda^2\int_L \varphi''(x_0)(x-x_0)\,d(x-x_0).$$

On the other hand, if λ is so small that $\|\lambda x_1\|<\frac{\alpha}{2}$, $\|\lambda x_2\|<\frac{\alpha}{2}$, then for the points of the contour L_λ we have $\|\rho(x-x_0)\|<\varepsilon\|x-x_0\|$ and the norm of the last integral in (7) is less than

$$\varepsilon(\|\lambda x_1\|+\|\lambda x_2\|)\,2(\|\lambda x_1\|+\|\lambda x_2\|)=2\varepsilon\lambda^2(\|x_1\|+\|x_2\|)^2.$$

But $\int_{L_\lambda} \varphi'(x_0)\,dx=0$, and thus we have

$$\lambda^2\Big\|\int_L \varphi''(x_0)(x-x_0)d(x-x_0)\Big\|\leqslant 2\varepsilon\lambda^2(\|x_1\|+\|x_2\|)^2,$$

whence, ε being arbitrary,

$$\int_L \varphi''(x_0)(x-x_0)\,d(x-x_0)=0.$$

If we denote by L^* the contour of the parallelogram having the points $0, x_1$ and x_2 as three of its vertices, it is easy to see that

$$\int_{L^*} \varphi''(x_0)\,x\,dx=0,$$

whence (according to III) follows the symmetry of the operation

$$u_2=\varphi''(x_0).$$

VI. We shall conclude this part of our discussion with the following theorem on the existence of a primitive function.

Theorem. *Let a function $\theta(x)$, with values belonging to the space $U=\{X\to Z\}$, be defined in a sphere $S\subset X$ and suppose that in S the function $\theta'(x)$ exists and that the values of $\theta'(x)$ [which evidently belong to the space $\{U\to Z\}=\{X, X\to Z\}$] are symmetrical operations; then there exists a function $\varphi(x)$ defined in S with values belonging to Z such that for any $x\in S'$ we have*

$$\varphi'(x)=\theta(x).$$

This function $\varphi(x)$ is determined up to an additive constant.

Proof. We shall prove first of all that the integral $\int \theta(x)\,dx$ does not depend on the path of integration. In fact, suppose the contrary, i. e. that there exists a triangle $L_0\subset S$ such that

$$\Big\|\int_{L_0}\theta(x)\,dx\Big\|=M>0.$$

Decompose this triangle L_0 into four equal triangles. Among these four triangles there exists at least one triangle L_1 such that

$$\Big\|\int_{L_1}\theta(x)\,dx\Big\|\geqslant\frac{M}{4}.$$

We shall decompose this triangle again into four equal triangles and denote by L_2 one of them with the property $\int_{L_2}\theta(x)\,dx\geqslant\frac{M}{16}$, and so on.

Thus, we obtain a sequence $L_0, L_1, \ldots$ of triangles, each of which lies inside the preceding one and such that for any positive integer n we have $|L_n| = \frac{1}{2^n}|L_0|$ and

$$\left\| \int_{L_n} \theta(x)\, dx \right\| \geqslant \frac{1}{2^{2n}} M.$$

The broken lines L_n approach (as n increases indefinitely) a certain point x_0. For n sufficiently large and $x \in L_n$ we shall have

$$\| \theta(x) - \theta(x_0) - \theta'(x_0)(x - x_0) \| = \| \rho(x - x_0) \| \leqslant \varepsilon \| x - x_0 \|.$$

Hence follows

$$\frac{1}{2^{2n}} M \leqslant \left\| \int_{L_n} \theta(x)\, dx \right\| = \left\| \int_{L_n} [\theta(x) - \theta(x_0) - \theta'(x_0)(x - x_0)]\, dx \right\| =$$

$$= \left\| \int_{L_n} \rho(x - x_0)\, dx \right\| \leqslant \varepsilon |L_n|^2 = \frac{\varepsilon}{2^{2n}} |L_0|^2,$$

i. e. $M \leqslant \varepsilon |L_0|^2$, whence ($\varepsilon$ being arbitrary) $M = 0$.

It is now evident that the function $\varphi(x) = \int_{x_0}^{x} \theta(x)\, dx$, where x_0 is an arbitrary point of S, is a primitive function for the function $\theta(x)$.

VII. This and the next § contain some results concerning the convergence of power series in Banach spaces. Let X and Z be two Banach spaces and let $u_0 \in Z$, $u_n \in [X_n \to Z]$ $(n = 1, 2, \ldots)$. Consider the series

$$\sum_{n=0}^{\infty} u_n x^n. \tag{8}$$

The domain of convergence A of such a series is a star, i. e. if a point $x \in A$ and if $-1 < t < 1$, then $tx \in A$.

We shall say that a point $x \in A$ is a point of uniform convergence, if there exists a neighbourhood of x, in which the series (8) converges uniformly. Denoting by $\underline{A}$ the set of all inner points of A and by B the set of all points of uniform convergence of the series (8), we have the following two theorems:

a) the set $A - \underline{A}$ is of the first category in X;

b) the set $\underline{A} - B$ is nowhere dense in X.

VIII. Theorem. If $x_0 \in B$, such a neighbourhood S of point x_0 exists that for every $x \in S$

$$\sum_{n=0}^{\infty} \sum_{k=0}^{n} \binom{n}{k} \| u_n x_0^{n-k} (x - x_0)^k \| < \infty.$$

Corollary 1. If $x \in S$, converges the series

$$\sum_{k=0}^{\infty} \left(\sum_{n=k}^{\infty} \binom{n}{k} u_n x_0^{n-k} \right) (x - x_0)^k.$$

Corollary 2. The series (8) may be differentiated term by term at every point $x_0 \in B$.

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Received
8. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MATHÉMATIQUES

SUR QUELQUES PROPRIÉTÉS DES PRODUITS SEMI-DIRECTS

Par P. KONTOROVITCH

(Présenté par M. I. Vinogradow, de l'Académie, le 28. I. 1939)

On introduit dans cet article la notion d'un groupe qui est un produit semi-direct de deux sous-groupes et on démontre certaines propriétés d'un tel produit. On, donne, d'ailleurs, une généralisation d'un théorème de A. A. Koulakoff (1).

§ 1. Un groupe G sera nommé produit semi-direct de ses sous-groupes F et H, si F est un diviseur normal, si le produit FH forme tout le groupe G, et si, d'ailleurs, F et H n'ont aucun facteur commun. Le produit semi-direct sera désigné par $G = F \succ H$; F sera nommé facteur direct, H—facteur complémentaire. Un cas particulier du produit semi-direct est le produit direct ordinaire $G = F \times H$.

Quels que soient deux systèmes A et B d'éléments du groupe G, on désigne par $N(A, B)$ le normalisateur de A par rapport à B, c'est-à-dire l'ensemble de tous les éléments du système B commutatifs avec le système A. D'une manière analogue $Z(A, B)$ est le centralisateur de A par rapport à B, c'est-à-dire l'ensemble de tous les éléments du système B commutatifs avec chacun des éléments du système A.

Il est évident que

$$N(A, B) = N(A) \cap B; \quad \text{où} \quad N(A) = N(A, G);$$
$$Z(A, B) = Z(A) \cap B; \text{ où} \quad Z(A) = Z(A, G).$$

Théorème. *Soit* $G = FH$; $\succ$ *alors*

$$N(H) = Z(H, F) \times H.$$

Démonstration.

1) $N(H) = HN(H, F)$. En effet, il suit de $hf \in N(H)$ et $h \in H$, $f \in F$ que $f \in N(H) \cap F = N(H, F)$, ou bien $N(H) \subset HN(H, F)$. D'autre part, il est évident que $N(H) \supset HN(H, F)$. Donc, $N(H) = HN(H, F)$.

2) $N(H, F)$ est un diviseur normal appartenant à $N(H)$. En effet, si $S \in N(H)$, on a $S^{-1}N(H, F)S \subset N(H) \cap F = N(H, F)$, et comme $S^{-1}N(H, F)S \simeq N(H, F)$, on a $S^{-1}N(H, F)S = N(H, F)$.

3) $N(H, F) = Z(H, F)$. En effet, H et $N(H, F)$ étant deux diviseurs normaux appartenant à $N(H)$ et n'ayant pas de facteur commun, leurs éléments sont commutatifs.

Donc, finalement, on a $N(H) = H \times Z(H, F)$.

§ 2. Un groupe Φ sera nommé groupe de Frobenius, s'il existe dans Φ un sous-groupe H ayant pour normalisateur $N(H) = H$ et tel que H est

sans facteur commun avec chacun de ses sous-groupes conjugués, c'est-à-dire $H \cap H^{(g)} = 1$, si g n'appartient pas à H, et $H = H^{(g)}$, si $g \in N(H) = H$, où $H^{(g)} = g^{-1}Hg, g \in \Phi$.

En vertu d'un théorème connu de Frobenius (3) il existe dans Φ un diviseur normal F tel que $F \cap H = 1$ et l'indice de H est égal à l'ordre de $F = y$. Il est évident que $\Phi = F \times H$, donc Φ est un produit semi-direct dont le facteur direct est F et le facteur complémentaire est H.

Lemme. *Si $\Phi = F \times H$ est un groupe de Frobenius, alors $N(h) \subset H$, où $h \neq 1$ est un élément arbitraire de H.*

Démonstration. Soit $a \in N(h)$, c'est-à-dire $a^{-1}ha = h \in H \cap H^{(a)} = H$, et ainsi $a \in H$.

Corollaire. Soit $\Phi = F \times H$ un groupe de Frobenius dont le facteur complémentaire H est abélien; alors $N(h) = H$, $h \neq 1 \in H$.

Théorème. *Soit $\Phi = F \times H$ un groupe de Frobenius dont le facteur complémentaire H est abélien. Soit $\Phi = F + Fh_2 + \ldots + Fh_m$ ($h_i \in H$). Désignons par $< h_i >$ la classe des éléments conjugués à h_i. Alors*

1) $< h_i > = Fh_i$, $h_i \neq 1$

2) $< h_i > < h_k > = < h_1 h_k >$ pour $h_1 h_k \neq 1$

et $< h_i > < h_k > = F$ pour $h_1 h_k = 1$.

Démonstration.

1) $h_i^{-1} g^{-1} h_i g = c(h, g)$ est le commutateur des éléments h_i et g, où g est un élément arbitraire de Φ. Il en résulte, le facteur du groupe $\frac{\Phi}{F} \simeq H$ étant abélien, que $h_i^{-1} g^{-1} h_i g = f \in F$, c'est-à-dire $g^{-1} h_i g \subset F$, ou bien $< h_i > \subset Fh_i$.

Mais * $O(< h_i >) = [G : N(h_i)] = (G : H) = O(Fh_i)$. Donc $< h_i > = Fh_i$.

2) $< h_i > = Fh_i$, $< h_k > = Fh_k$. Il en résulte $< h_i > < h_k > = Fh_i \times Fh_k = Fh_i h_k = < h_i h_k >$ pour $h_i h_k \neq 1$ et $Fh_i h_k = F$ pour $h_i h_k = 1$.

Corollaire. Si $\Phi = F \times H$ est un groupe de Frobenius à facteur complémentaire H, alors

$$h^k g^{-1} h^i g \in < h^{i+k} > \quad \text{pour} \quad i + k \neq 0$$

$$\text{et } h^k g^{-1} h^i g \in F \quad \text{pour} \quad i + k = 0;\ h \in H, g \in G.$$

Remarque. Le corollaire obtenu est une généralisation du théorème suivant de A. A. Koulakoff (2).

Soit $H = \{h\}$ un sous-groupe cyclique d'ordre p (p étant un nombre premier) d'un groupe G à normalisateur $N(H) = H$; alors h^{i+1} est conjugué à $hg^{-1}h^i g$, $h \in H$, $g \in G$, $i + 1 \neq 0$.

§ 3. Théorème. *Soit $G = F \times H$ et $N(h) = H$ pour chaque élément $h \neq 1$ de H; alors $G = \Phi$ est un groupe de Frobenius à facteur complémentaire abélien H.*

Démonstration. Il suit de $N(h) = H$ que H est un groupe abélien.

Démontrons que $N(H) = H$. Nous avons $N(H) = H \times Z(H, F)$, où $Z(H, F) = Z(h_1, F) \cap Z(h_2, F) \cap \ldots \cap Z(h_m, F)$; $m = O(H)$. Mais $Z(h_i, F) = N(h_i, F) = 1$. Il en résulte que $N(H) = H$.

Démontrons que H est sans facteur commun avec tous ses sous-groupes conjugués qui ne coïncident pas avec H. En effet, on a $G = F + Fh_2 + \ldots + Fh_m$.

En appliquant le même raisonnement que dans la démonstration du théorème du § 2, on trouve que $< h_i > = Fh_i$ pour $h_i \neq 1$.

---

* $O(A)$ désigne l'ordre, c'est-à-dire le nombre des éléments du système A.

Soit maintenant $H \cap g^{-1}Hg \neq 1$, $H \neq g^{-1}Hg$. On a alors $h_1 = g^{-1}h_2 g$, où g n'appartient pas à H, $h_1 \in H, h_2 \in H$.

Il en résulte que h_1, h_2 sont des éléments d'une même classe $<h_1> = Fh_1$, c'est-à-dire $h_2 = fh_1$, $f \in F$, ou bien $h_2 h_1^{-1} = f = 1$, c'est-à-dire $h_2 = h_1$. Donc, $h_1 = g^{-1}h_1 g$, c'est-à-dire $g \in N(h_1) = H$, ce qui contredit à l'hypothèse faite.

Ainsi G vérifie les conditions caractéristiques pour un groupe de Frobenius Φ à facteur complémentaire abélien.

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Manuscrit reçu
le 31. I. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MECHANICS

ON THE STATE OF STRESS OF A STRETCHED AND TWISTED BAR

By N. ZVOLINSKY

(Communicated by S. A. Tchaplyguin, Member of the Academy, 31. I. 1939)

In one of the preceding papers (1), published by the author together with P. Riz, was found the first approximation for the solution of the problem of torsion of a bar stretched by forces applied to its ends. In certain applications a similar problem, but under the assumption of mass tensile forces, is of interest. We give below the solution of such a problem in the first approximation of mass forces of constant intensity.

As a basis for this solution we take the exact relations between the components and the first derivatives of the displacements, and the relations between the stress components and the deformations true up to the terms of the second order inclusively (2):

$$\left.\begin{aligned} &\sigma_{xx} = \lambda I_1 + [2\mu + (\lambda - 2\mu) I_1]\, \varepsilon_{xx} + \frac{\lambda}{2} I_1^2 - 3\lambda I_2 + 5\mu(\varepsilon_{xx}^2 + \varepsilon_{xy}^2 + \varepsilon^2{}_{xz}) \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ &\sigma_{xy} = [2\mu + (\lambda - 2\mu) I_1]\, \varepsilon_{xy} + 5\mu\, [\varepsilon_{xy}(\varepsilon_{xx} + \varepsilon_{yy}) + \varepsilon_{xz}\varepsilon_{yz}] \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{aligned}\right\} \quad (1)$$

By $\varepsilon_{xx}, \varepsilon_{yy}, \dots, \varepsilon_{zz}$ are denoted the deformation components referred to rectangular Cartesian co-ordinates, and by $\sigma_{xx}, \sigma_{xy}, \dots, \sigma_{zz}$ the corresponding stress components. Here, as well as formerly, we had to assume, for want of more precise experimental data, the existence of some finite interval, within which we have at a tension in three directions linear relation between stresses and elongations.

Let us denote by a, b, c the co-ordinates of a point before the deformation and by x, y, z — the co-ordinates of the same point after the deformation. These co-ordinates are connected by means of the displacement components u, v, w:

$$x = a + u, \quad y = b + v, \quad z = c + w.$$

The quantities (components, stresses and deformations) referring to the deformed state we shall supply with the indices x, y, z.

Let us place the origin of the system of co-ordinates in the centre of gravity of one of the bases of the cylinder, and direct the axis Oz

along the axis of the cylinder, and the axes Ox and Oy — along the principal axes of the cross-section. We confine ourselves for the sake of simplicity to the case of a section having two axes of symmetry and consider the problem of determining the state of stress of the cylinder under the following conditions:

a) the points of the cylinder are subjected to mass load, the intensity of which is q and the direction is parallel to the Oz axis;

b) the lateral surface is free from stresses;

c) on the butt surface, which before the deformation corresponded to the plane $c=0$, the system of external forces is statistically equivalent to the twisting couple with the moment M.

This formulation of the problem involves an element of indefiniteness, but for the determination of the stressed state far from the ends of the cylinder it is sufficient, by St. Venant's principle, to find some particular solution of the formulated problem.

This solution we find by developing the displacements into a power-series in the small parameters α and τ:

$$\left.\begin{aligned} u &= -\tau bc - \alpha\nu ac + \alpha\tau u_1 + \alpha^2 u_2; \\ v &= \tau ac - \alpha\nu bc + \alpha\tau v_1 + \alpha^2 v_2; \\ w &= \tau\varphi(a, b) + \frac{a}{2}[c^2 + \nu a^2 + \nu b^2] + \alpha\tau w_1 + \alpha^2 w_2. \end{aligned}\right\} \qquad (2)$$

The functions u_1, v_1, w_1, as well as u_2, v_2, w_2, are to be determined; $\varphi(a, b)$ is the torque function, ν — the coefficient of Poisson, the parameters α and τ have the values:

$$\tau = \frac{M}{\mu T_0}; \quad \alpha = \frac{q}{2\mu(1+\nu)},$$

where T_0 is the geometrical torque stiffness. The terms with τ^2 in the expressions (2) are omitted which shows the smallness of the torsion.

If we denote by $\sigma'_{xx}, \sigma'_{xy}, \ldots, \sigma'_{zz}$ the stresses derived from the displacements u_1, v_1, w_1 according to the classical formulae and by $\sigma''_{xx}, \sigma''_{xy}, \ldots, \sigma''_{zz}$ the stresses derived similarly from the displacements u_2, v_2, w_2, then for the stresses of the formulated non-linear problem we shall have with the assumed precision:

$$\left.\begin{aligned} \sigma_{xx} &= \alpha\tau\lambda[(1-2\nu)ab + \nu(a\varphi_a + b\varphi_b)] + \alpha^2\frac{\lambda\nu}{2}(a^2 + 2\nu b^2) + \\ &\quad + \alpha\tau\sigma'_{xx} + a^2\sigma''_{xx}; \\ \sigma_{yy} &= -\alpha\tau\lambda[(1-2\nu)ab - \nu(a\varphi_a + b\varphi_b)] + \alpha^2\frac{\lambda\nu}{2}(2\nu a^2 + b^2) + \\ &\quad + \alpha\tau\sigma'_{yy} + \alpha^2\sigma''_{yy}; \\ \sigma_{zz} &= \alpha 2\mu(1+\nu)c + \alpha\tau\lambda(1-\nu)(a\varphi_a + b\varphi_b) + \\ &\quad + \alpha^2\nu\left(\mu c^2 + \frac{\lambda}{2}(a^2 + b^2)\right) + \alpha\tau\sigma'_{zz} + \alpha^2\sigma''_{zz}; \\ \sigma_{xy} &= \alpha\tau\mu\nu(b^2 - a^2) + \alpha^2\mu\nu^2 ab + \alpha\tau\sigma'_{xy} + \alpha^2\sigma''_{xy}; \\ \sigma_{yz} &= \tau\mu(\varphi_b + a) + \alpha\tau\frac{\mu c}{2}[(3\nu - 1)\varphi_b + (7\nu + 1)a] - \\ &\quad - \alpha^2\mu\nu(1+\nu)bc + \alpha\tau\sigma'_{yz} + \alpha^2\sigma''_{yz}; \\ \sigma_{zx} &= \tau\mu(\varphi_a - b) + \alpha\tau\frac{\mu c}{2}[(3\nu - 1)\varphi_a - (7\nu + 1)b] - \\ &\quad - \alpha^2\mu\nu(1+\nu)ac + \alpha\tau\sigma'_{zx} + \alpha^2\sigma''_{zx}. \end{aligned}\right\} \qquad (3)$$

For the determination of the stresses $\sigma'_{xx}, \ldots, \sigma'_{zz}$ we obtain the system of equations:

$$\left.\begin{aligned}
&\frac{\partial \sigma'_{xx}}{\partial a}+\frac{\partial \sigma'_{xy}}{\partial b}+\frac{\partial \sigma'_{xz}}{\partial c}=\frac{\mu}{2}(1+\nu)\,b+\frac{\mu}{2}\,\frac{1-5\nu+2\nu^2}{1-2\nu}\,\varphi_a-\frac{\lambda}{2}(a\varphi_{aa}+b\varphi_{ab});\\
&\frac{\partial \sigma'_{xy}}{\partial a}+\frac{\partial \sigma'_{yy}}{\partial b}+\frac{\partial \sigma'_{yz}}{\partial c}=-\frac{\mu}{2}(1+\nu)\,a+\frac{\mu}{2}\,\frac{1-5\nu+2\nu^2}{1-2\nu}\,\varphi_b-\\
&\qquad\qquad -\frac{\lambda}{2}(a\varphi_{ab}+b\varphi_{bb});\\
&\frac{\partial \sigma'_{zx}}{\partial a}+\frac{\partial \sigma'_{zy}}{\partial b}+\frac{\partial \sigma'_{zz}}{\partial c}=0,
\end{aligned}\right\}\qquad (4)$$

and the boundary conditions:

$$\left.\begin{aligned}
&\sigma'_{xx}\cos(na)+\sigma'_{xy}\cos(nb)+\lambda\left[(1-\nu)\,a\varphi_a+\nu b\varphi_b\right]\cos(na)+\\
&\qquad +\mu\nu\left[a\varphi_b+b\varphi_a\right]\cos(nb)=0;\\
&\sigma'_{yx}\cos(na)+\sigma'_{yy}\cos(nb)+\mu\nu\,(a\varphi_b+b\varphi_a)\cos(na)+\\
&\qquad +\lambda\left[(1-\nu)\,b\varphi_a+\nu a\varphi_a\right]\cos(nb)=0;\\
&\sigma'_{zx}\cos(na)+\sigma'_{zy}\cos(nb)+\mu c\left[b\cos(na)-a\cos(nb)\right]=0,
\end{aligned}\right\}\qquad (4')$$

where $\cos(na)$, $\cos(nb)$ are the direction cosines of the original cylindric surface.

Moreover, the stresses $\sigma'_{xx}, \ldots, \sigma'_{zz}$ have to satisfy the classical conditions of compatibility. The stresses $\sigma''_{xx}, \ldots, \sigma''_{zz}$ are determined from the equilibrium equations:

$$\left.\begin{aligned}
&\frac{\partial \sigma''_{xx}}{\partial a}+\frac{\partial \sigma''_{xy}}{\partial b}+\frac{\partial \sigma''_{xz}}{\partial c}=\frac{\lambda}{2}(1-4\nu)\,a;\\
&\frac{\partial \sigma''_{yx}}{\partial a}+\frac{\partial \sigma''_{yy}}{\partial b}+\frac{\partial \sigma''_{yz}}{\partial c}=\frac{\lambda}{2}(1-4\nu)\,b;\\
&\frac{\partial \sigma''_{zx}}{\partial a}+\frac{\partial \sigma''_{zy}}{\partial b}+\frac{\partial \sigma''_{zz}}{\partial c}=2\mu(1+\nu+\nu^2)\,c,
\end{aligned}\right\}\qquad (5)$$

the compatibility conditions and the following boundary conditions:

$$\left.\begin{aligned}
&\sigma''_{xx}\cos(na)+\sigma''_{xy}\cos(nb)+\frac{\lambda\nu}{2}(a^2+2\nu b^2)\cos(na)+\mu\nu^2 ab\cos(nb)=0;\\
&\sigma''_{yx}\cos(na)+\sigma''_{yy}\cos(nb)+\mu\nu^2 ab\cos(na)+\frac{\lambda\nu}{2}(2\nu a^2+b^2)\cos(nb)=0;\\
&\sigma''_{zx}\cos(na)+\sigma''_{zy}\cos(nb)+\mu\nu(1+\nu)\,c\left[a\cos(na)+b\cos(nb)\right]=0.
\end{aligned}\right\}\qquad (5')$$

Let us denote by I_p the polar moment of inertia of the cross-section and put $\omega=\frac{I_p}{T_0}$. Then, solving the problems (4), (4′) and (5), (5′), we find for the full stresses:

$$\left.\begin{aligned}
&\sigma_{xx}=\alpha\tau E\left\{\frac{\nu}{1+\nu}\,a\left(b-\frac{\partial\varphi}{\partial a}\right)+(\omega-1)\,ab-\omega\varphi+\frac{\partial^2 F}{\partial b^2}\right\}-\alpha^2\frac{\partial^2\Phi}{\partial b^2};\\
&\sigma_{yy}=\alpha\tau E\left\{-\frac{\nu}{1+\nu}\,b\left(a+\frac{\partial\varphi}{\partial b}\right)-(\omega-1)\,ab-\omega\varphi+\frac{\partial^2 F}{\partial a^2}\right\}-\alpha^2\frac{\partial^2\Phi}{\partial a^2};\\
&\sigma_{zz}=\alpha Ec+\alpha\tau E\left\{\frac{\nu}{1+\nu}\left(a\frac{\partial\varphi}{\partial a}+b\,\frac{\partial\varphi}{\partial b}\right)+\nu\nabla^2 F-\right.\\
&\qquad\left.-\left(2\omega-\frac{1+3\nu}{2}\right)\varphi+A'a+B'b+C'\right\}+\alpha^2\left\{\mu\nu^2(3+2\nu)\times\right.\\
&\qquad\left.\times\left(a^2+b^2-\frac{I_p}{S}\right)+\nabla^2\Phi+\mu(1+\nu)\,c^2\right\};\\
&\sigma_{xy}=\alpha\tau E\left\{\frac{\nu}{2(1+\nu)}\left[\,b\left(b-\frac{\partial\varphi}{\partial a}\right)-a\left(a+\frac{\partial\varphi}{\partial b}\right)\right]-\frac{\partial^2 F}{\partial a\,\partial b}\right\}+\alpha^2\frac{\partial^2\Phi}{\partial a\,\partial b};\\
&\sigma_{yz}=\tau\mu\left(\frac{\partial\varphi}{\partial b}+a\right)+\alpha\tau Ec\left\{\left(\frac{3\nu}{2(1+\nu)}-\omega\right)\left(\frac{\partial\varphi}{\partial b}+a\right)+a\right\};\\
&\sigma_{zx}=\tau\mu\left(\frac{\partial\varphi}{\partial a}-b\right)+\alpha\tau Ec\left\{\left(\frac{3\nu}{2(1+\nu)}-\omega\right)\left(\frac{\partial\varphi}{\partial a}-b\right)-b\right\}.
\end{aligned}\right\}\qquad (6)$$

In these formulae $F(a, b)$ denotes the function satisfying in the domain of the cross-section the biharmonic equation

$$\nabla^4 F = 0$$

and on its contour the conditions:

$$\frac{\partial F}{\partial a} = -\int_0^s [(\omega - 1)\, ab + \omega\varphi]\, da,$$

$$\frac{\partial F}{\partial b} = \int_0^s [(\omega - 1)\, ab - \omega\varphi]\, db.$$

$\Phi(a, b)$ denotes the function satisfying in the domain of the cross-section the equation

$$\nabla^4 \Phi = 8\varkappa, \quad \varkappa = \frac{\mu\nu^2}{4(1-\nu^2)}(1 - 5\nu - 4\nu^2)$$

and on its boundary the conditions:

$$\Phi = 0; \quad \frac{\partial \Phi}{\partial n} = 0,$$

where $\frac{\partial}{\partial n}$ is the normal derivative on the contour.

The quantities A', B', C' in the formulae (6) are constants determined by the condition that the projection on the Oz axis of the principal vector of external forces, applied to the free butt surface, is equal to zero, and that the moments of these forces with respect to the Ox and Oy axes are also equal to zero.

From the formulae (6) we can conclude that the supplementary stresses obtained as the result of introduction of non-linear terms are, generally speaking, very small, since their order of magnitude, respectively the main stresses, can be expressed by the number αl, where

$$\alpha l = \frac{ql}{2\mu(1+\nu)} = \frac{\sigma}{E} \leqslant \frac{\text{limit of proportionality}}{\text{Young's modulus}}.$$

However, there may occur cases, in which the supplementary stresses will be comparable with the main stresses. This will, e. g., happen when the quantity ω occurring in (6) becomes large. This circumstance may occur in the case of thin bars, for which ω can be made arbitrarily large by a corresponding reduction of the thickness.

In concluding thanks are due to P. M. Riz for the discussion of some problems.

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Received
1. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MECHANICS

ON THE FLEXURE OF A STRETCHED PRISMATIC BAR

By P. M. RIZ

(Communicated by S. Tchaplyguin, Member of the Academy, 31.I.1939)

In several technical problems, as, for instance, in the calculation of air-screw blades, it is necessary to consider the flexure of a bar considerably stretched along its axis. The problem of simultaneous action of loads of different character and of the interaction of deformations caused by them cannot be solved within the scope of linear theory of elasticity, in which the principle of superposition excludes the possibility of such interaction.

The present paper uses the methods of a non-linear theory of elasticity exposed in our common paper with N. Zvolinsky (1). We shall require the following results:

1. The deformation is determined by the components of the tensor of deformations referred to the deformed state, according to the formulae introduced by Filon:

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} - \frac{1}{2}\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right\} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \frac{1}{2}\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right). \end{aligned}\right\} \qquad (1)$$

2. For deformations which are so small that the linear dependence between the main stresses and the main elongations is not disturbed, the deformation components are connected with the stresses by the following formulae:

$$\left.\begin{aligned} \sigma_{xx} &= \lambda I_1 + [2\mu + (\lambda - 2\mu) I_1]\,\varepsilon_{xx} + \frac{\lambda}{2} I_1^2 - 3\lambda I_2 + 5\mu\,(\varepsilon_{xx}^2 + \varepsilon_{xy}^2 + \varepsilon_{xz}^2) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \sigma_{xy} &= [2\mu + (\lambda - 2\mu) I_1]\,\varepsilon_{xy} + 5\mu\,[\varepsilon_{xy}\,(\varepsilon_{xx} + \varepsilon_{yy}) + \varepsilon_{xz}\,\varepsilon_{yz}] \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}\right\} \qquad (2)$$

where I_1 and I_2 are the invariants of the tensor of deformations.

3. The equations of equilibrium involving the stresses preserve their classical form with the only difference that the derivatives are taken with respect to the co-ordinates of the final state.

We shall consider a prismatic bar of length l* fixed in one cross-section, to the free end of which are applied loads parallel to the axis of the bar, which amount to a tension force P and a flexing couple M_0. Let a, b and c be the Cartesian co-ordinates of the points of the bar in the undeformed state. We assume the following expressions for the displacement:

$$\left.\begin{aligned} u &= -\alpha\nu a + \frac{\beta}{2}[c^2 + \nu(a^2 - b^2)] + \alpha^2 u^{(1)} + \alpha\beta u^{(2)}, \\ v &= -\alpha\nu b + \beta\nu ab + \alpha^2 v^{(1)} + \alpha\beta v^{(2)}, \\ w &= \alpha c - \beta ac + \alpha^2 w^{(1)} + \alpha\beta w^{(2)}, \end{aligned}\right\} \tag{3}$$

which represent the superposition of displacements due to tension and to pure flexure with the addition of some supplementary displacements of a higher order of smallness. Herein

$$\alpha = \frac{P}{ES} \text{ and } \beta = -\frac{M_0}{EJ},$$

and the constant β is supposed to be small in the classical sense, whereas we shall not assume the same with respect to the quantity α. We shall everywhere retain the terms of the orders $\alpha\beta$ and α^2 and neglect the terms of a higher order of smallness.

This formulation of the problem corresponds to small deformations due to pure flexure accompanied by considerable tension along the axis.

The co-ordinate c is reckoned along the axis of the bar, a — in the direction of the flexure effected in the principal plane and b — perpendicularly to the first two axes. The corresponding co-ordinates in the deformed state are z, x and y.

According to (1) and then (2), we find the stresses:

$$\left.\begin{aligned} \sigma_{xx} &= \alpha^2\sigma_{xx}^{(1)} + \alpha\beta\sigma_{xx}^{(2)}, \\ \sigma_{yy} &= \alpha^2\sigma_{yy}^{(1)} + \alpha\beta\sigma_{yy}^{(2)}, \\ \sigma_{zz} &= -E(-\alpha + \beta a) + 2\nu E(\alpha^2 - 2\alpha\beta a) + \alpha^2\sigma_{zz}^{(1)} + \alpha\beta\sigma_{zz}^{(2)}, \\ \sigma_{xy} &= \alpha^2\sigma_{xy}^{(1)} + \alpha\beta\sigma_{xy}^{(2)}, \\ \sigma_{xz} &= \frac{E}{2}\alpha\beta c + \alpha^2\sigma_{xz}^{(1)} + \alpha\beta\sigma_{xz}^{(2)}, \\ \sigma_{yz} &= \alpha^2\sigma_{yz}^{(1)} + \alpha\beta\sigma_{yz}^{(2)}. \end{aligned}\right\} \tag{4}$$

Let us now write the equation of equilibrium and express the condition that the lateral surface of the bar is free from stresses, as well as the condition at the butt end, which must be satisfied in order that the stresses should be reducible to an axial force P and to a flexing couple of the moment M_0.

Demanding that these conditions should be identically satisfied for arbitrary values of the parameters α and β, we come to the following boundary problems for the determination of the supplementary stresses.

1) The problem for determination of $\sigma_{xx}^{(1)}$ etc. The equations of equilibrium:

$$L_1^{(1)} = 0, \quad L_2^{(1)} = 0, \quad L_3^{(1)} = 0.$$

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* We shall not make here assumptions on the smallness of all the linear dimensions of the cross-sections of the bar in comparison with its length, in the sense of Kirchhoff.

The conditions on the lateral surface:

$$M_1^{(1)}=0, \quad M_2^{(1)}=0, \quad M_3^{(1)}=0.$$

Herein

$$L_1^{(1)}=\frac{\partial\sigma_{xx}^{(1)}}{\partial a}+\frac{\partial\sigma_{xy}^{(1)}}{\partial b}+\frac{\partial\sigma_{xz}^{(1)}}{\partial c},$$

$$M_1^{(1)}=\sigma_{xx}\cos\nu x+\sigma_{xy}\cos\nu y+\sigma_{xz}\cos\nu z.$$

On the butt surface:

$$2\nu ES+\int\int\sigma_{zz}^{(1)}\,dadb=\int\int a\sigma_{zz}^{(1)}\,dadb=\int\int b\sigma_{zz}^{(1)}\,dadb=0.$$

2) The problem for determination of $\sigma_{xx}^{(2)}$ etc.

$$L_1^{(2)}=-\frac{E}{2}, \quad L_2^{(2)}=0, \quad L_3^{(2)}=0,$$

$$M_1^{(2)}=0, \quad M_2^{(2)}=0, \quad M_3^{(2)}=\frac{E}{2}c\frac{\partial f}{\partial a},$$

$f(a, b)=0$ is the equation of the undeformed lateral surface.

By means of the substitution

$$\sigma_{xz}^{(2)}=\frac{E}{2}c+\bar{\sigma}_{xz}^{(2)}$$

this problem may be easily reduced to the problem of flexure of a bar under the action of its own weight with the equations:

$$\bar{L}_1^{(2)}=-E, \bar{L}_2^{(2)}=0, \quad \bar{L}_3^{(2)}=0,$$
$$\bar{M}_1^{(2)}=0, \quad \bar{M}_2^{(2)}=0, \quad \bar{M}_3^{(2)}=0,$$

and the conditions at the butt:

$$\int\int\bar{\sigma}_{xz}^{(2)}\,dadb+Els=0; \quad \int\int a\bar{\sigma}_{zz}^{(2)}\,dadb=0.$$

This last problem may be in its turn easily reduced to a plane problem; for the case of an elliptical cross-section a particular solution is known (it does not, however, satisfy the conditions in the free cross-section), which belongs to Pearson and Filon [2]. We give the further calculations for this particular case, using the results of the paper just referred to and changing them only in accordance with the conditions at the butt. Here are the final expressions for the stresses:

$$\left.\begin{aligned}
\sigma_{xx}&=\alpha\beta\frac{2\mu}{3A^2}\left\{(A^2-a^2)k_1a-k_2ab^2\right\};\\
\sigma_{yy}&=\alpha\beta\frac{2\mu}{3A^2}\left\{(B^2-b^2)k_3a-k_4a^3\right\};\\
\sigma_{xy}&=\alpha\beta\frac{2\mu}{3A^2}\left\{(B^2-b^2)k_5b-k_6a^2b\right\};\\
\sigma_{yz}&=\alpha\beta\frac{8\mu}{A^2}k_7(c+m_1)ab;\\
\sigma_{xz}&=\alpha\beta\left\{Ec+\frac{2\mu(c+m_1)}{A^2}\left[(a^2-A^2)k_8-b^2k_9\right]\right\};\\
\sigma_{zz}&=E\alpha-\beta Ea+\alpha\beta\left\{\frac{2E}{A^2}(l^2-c^2)-\frac{\mu}{3}k_{10}-4\nu E-\frac{4E}{A^2}m_1c+\right.\\
&\qquad\left.+m_2-(a^2-3b^2)\frac{2\mu}{A^2}k_{11}+\frac{4\mu}{3A^2}a^2(2+\nu)\right\}a.
\end{aligned}\right\}\quad(5)$$

A and B are the half-axes of the ellipse, $\varepsilon = \frac{B}{A}$, k_i and m_i are certain constants, whose values we need not state here.

The asymptotical expressions of these formulae for $\varepsilon \to \infty$ are:

$$\left.\begin{aligned}
\sigma_{xx} &= 0,\\
\sigma_{yy} &= \alpha\beta\,(B^2 - b^2)\,\frac{2\mu}{5A^2}\,(7\nu - 1)\,a,\\
\sigma_{xy} &= -\alpha\beta\,\frac{2\mu}{5A^2}\,(1 + 3\nu)\,\varepsilon^2 a^2 b,\\
\sigma_{xz} &= \alpha\beta\left\{Ec + \frac{4\mu}{A^2}\,a^2\left(c - l\,\frac{1-2\nu}{2-\nu}\right) + \frac{4\mu(1-2\nu)}{B^2}\,b^2\left(c - l\,\frac{1-2\nu}{2-\nu}\right)\right\},\\
\sigma_{yz} &= \alpha\beta\,\frac{8\mu\nu}{A^2}\left(c - l\,\frac{1-2\nu}{2-\nu}\right),\\
\sigma_{zz} &= E\alpha - \rho E a + \alpha\beta\left\{\frac{2E}{A^2}\,(l^2 - c^2) - \frac{4E}{A^2}\,l\,(l-c)\,\frac{1-2\nu}{2-\nu} + \right.\\
&\quad \left. + \frac{2\mu}{A^2}\,(11\nu + 3\nu^2)\left(3b^2 - \frac{B^2}{15}\right)\right\} a.
\end{aligned}\right\} \quad (6)$$

In the usual calculations of flexed and stretched bars infinitely thin in the sense of Kirchhoff, the only determinable — and this under certain hypotheses — stress is σ_{zz}.

The corresponding formula in our denotations will have the form:

$$\sigma_{zz} = E\alpha - \beta E a + \alpha\beta\,\frac{2E}{A^2}\,(l^2 - c^2)\,a. \quad (7)$$

In spite of the practical smallness of the quantities α and β, it is easily seen that if A is very small in comparison with l and B, the stresses obtained from formulae (6) will be quite considerable and also the value of the stress σ_{zz} will noticeably differ from that given by the formula (7).

The displacements may be easily obtained by the usual methods and we refrain from giving them.

In conclusion I should like to express my profound gratitude to N. Zvolinsky for his valuable advices.

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Received
1.II.1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PHYSICS

THEORY AND DESIGN OF THE SIMPLEST RESONANCE SOUND-ABSORBING SYSTEMS

By S. N. RSCHEVKIN

(Communicated by S. I. Wawilow, Member of the Academy, 29. I. 1939)

I. Monoresonance or One-Layer Systems. The author has theoretically proved the possibility of obtaining high absorption of sound with the help of a system of resonators located over the surface of a wall (1). In the case of the normal incidence of sound on the system of resonators, arranged in the form of regular lattice work over the surface of a sufficiently large wall, we are entitled, owing to symmetry, to consider the whole process as the propagation of flat waves moving in pipes with rigid walls comprising one cell of the system of resonators. From this point of view, it is immaterial whether or not walls will exist between the separate resonators. With an oblique incidence the absence or the presence of walls substantially changes the course of the process, and the question calls for special consideration.

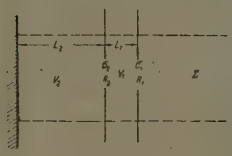

Fig. 1.

In the case of the absence of walls the simplest resonance system may be designed in the form of a metallic sheet with equispaced apertures (one aperture of area σ per surface $\sum$ of wall) lying at a certain distance L from the wall. Into every aperture porous material should be introduced giving a desired friction; with small diameter of the apertures, considerable friction will be available even without the introduction of porous material. We shall denote the simplest monolayer system as a monoresonance sound-absorbing system. With two and respectively with three perforated screens we obtain biresonance (double-layer) (Fig. 1) and triresonance (three-layer) systems. With a large number of perforated screens the system can be generally spoken of as a polyresonance or polylayer system.

The absorption coefficient of any resonance sound-absorbing system with normal incidence is determined (1) from the expression

$$\alpha = \frac{4R'}{(R'+1)^2 + Y'^2}, \qquad (1)$$

where quantities R' and Y' are bound with the active (R) and reactive (Y) components of the input impedance at the opening of the system ($Z = R + jY$) by relations:

$$R' = \frac{R}{W} \quad \text{and} \quad Y' = \frac{Y}{W}, \qquad (2)$$

where $W=\frac{\sigma^2}{\Sigma}\rho c=\frac{\sigma^2}{\Sigma^2}\sum\rho c$ represents the radiation resistance of a pipe with cross-section $\sum$ (ρ — density of air, c — sound velocity) transformed to the section σ of the openings of the resonators. The quantity $R'\rho c$ gives resistance (averaged) per unit of surface of the wave front.

The quantity R in the case of a monosystem is determined only by friction in aperture σ, as well as by other losses in the resonator; quantity Y is a function of frequency, which involves, as its parameters, added mass M in the aperture σ, as well as elasticity E of air volume V;

$$Y=M\omega-\frac{E}{\omega}=\frac{\rho c^2}{K}\omega-\frac{\rho c^2\sigma^2}{V\omega}=W\left[\frac{\Sigma}{cK}\omega-\frac{c\Sigma}{V\omega}\right]=WY', \qquad (3)$$

where $\omega=2\pi f$ is the angular frequency of sound and K — the so-called conductivity of aperture σ. For an aperture of diameter D in a thin screen $K\cong D$; for a pipe with length l $K=\frac{\sigma}{l}$ (2).

It will be possible to determine all the dimensions of the system, if we let a certain minimum absorption coefficient α at boundaries f_1 and f_2 of a certain interval of frequencies, within which the resonance maximum of absorption is to be found.

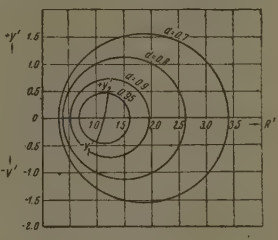

Fig. 2.

As will be easily seen from (1), the points corresponding to $\alpha=\alpha_1$, in the plane of the complex variable R', Y', lie on the circle of radius $\frac{2\sqrt{1-\alpha_1}}{\alpha_1}$, whose centre is on axis R' at a distance $\frac{2-\alpha_1}{\alpha_1}$ from the origin. With the value of $Y'=0$, α obtains the value of $\alpha_{max}=\frac{4R'}{(R'+1)^2}$. The circles of equal α are shown in Fig. 2. The points corresponding to various frequencies lie on a certain curve in plane R', Y' and its intersection with circumference α_1 gives points Y_1' and Y_2' (with frequencies f_1 and f_2), with the minimum values of $\alpha=\alpha_1$, at the sides of maximum α_{max}. For these points on the basis of (3) we obtain two conditions:

$$\frac{\Sigma}{cK}\omega_1-\frac{c}{L\omega_1}=-Y_1', \quad \frac{\Sigma}{cK}\omega_2-\frac{c}{L\omega_2}=+Y_2' \qquad (4)$$

where $L=\frac{V}{\Sigma}$ is the depth of the resonance layer. These conditions may be regarded as two linear equations relating to unknowns $\frac{\Sigma}{K}$ and $\frac{1}{L}$, which are readily solved.

Knowing the law of the dependence of friction R upon frequency f, it will be possible to select the most advantageous $R_0'=\frac{1}{W}R(f)_{f=f_0}$ at the resonance and to find points Y_1' and Y_2'. Experience has, however, shown that friction may be regarded as independent of frequency, while additional mass is barely a function of diameter and length of neck and is independent of friction. In this case curve Y_1', Y_2' is transformed into a straight line, parallel to axis Y'. It will be reasonable to choose the value of R_0' at the resonance lying in the centre of circle α_1, in order to obtain $\alpha>\alpha_1$ in the largest possible range of frequencies; then $Y_1'=Y_2'$

will be equal to the radius of a circle corresponding to α_1, and in accordance with the above-mentioned we shall obtain from (4):

$$L = \frac{c\alpha_1}{\sqrt{1-\alpha_1}} \frac{\omega_2 - \omega_1}{\omega_1\omega_2} = \frac{c\alpha_1}{2\pi\sqrt{1-\alpha_1}} \frac{1-\frac{f_1}{f_2}}{f_1} = \frac{\alpha}{2\pi\sqrt{1-\alpha_1}}(\lambda_1 - \lambda_2), \quad (5)$$

$$\frac{\Sigma}{K} = \frac{c\sqrt{1-\alpha_1}}{\alpha_1(\omega_2-\omega_1)} = \frac{c\sqrt{1-\alpha_1}}{2\pi\alpha_1 f_1} \frac{1}{\frac{f_2}{f_1}-1}. \quad (6)$$

It will be easy to see that Y' (3) with given α_1 and f_2/f_1 will depend solely upon relation f/f_1. Thus, when plotting a certain curve for α on a logarithmical scale we obtain a curve for another initial frequency f_1', simply by shifting upon an interval $q = f_1'/f_1$ along the axis of frequencies. The respective values L and $\frac{\Sigma}{K}$ will in conformity with (5) and (6) be found to be q times less.

For obtaining absorption $\alpha > 0.9$ within the range of frequencies between 100—5000 Hz, it would have been necessary, in accordance with (5), to take $L = 76.5$ cm. Under these conditions the resonators would turn into long pipes and it would be no longer correct to assume their elasticity as being equal to $E = \frac{\rho c^2 \sigma^2}{V} = W\frac{c}{L}$, as for a concentrated volume. As it follows from the theory of sound propagation in a pipe, we have to take in this case instead of (3) the following expression:

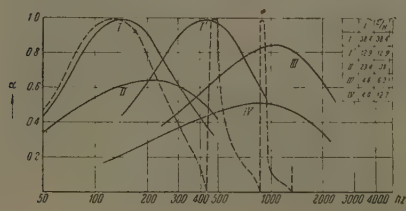

Fig. 3.

$$Y = W\left[\frac{\Sigma}{cK}\omega - \operatorname{ctg}\frac{L}{c}\omega\right]. \quad (7)$$

With $\frac{L}{c}\omega = n\pi$ or with $f_k = n\frac{c}{2L}$, we obtain $Y' = -\infty$ and $\alpha = 0$. In this case along the length of the pipe quite a number of standing semi-waves can be ranged. The velocity knot is formed in aperture σ and the pipe is, as it were, shut up to sound and therefore $\alpha = 0$. For the case in hand when $L = 76.5$ cm, we obtain $f_k = n.222$ Hz. With these frequencies there arise in curve α drops running down to zero; between them will lie maximums with values of $\alpha = 0.99$. Thus, the obtaining of $\alpha > 0.9$ within a broad range with the help of resonance system encounters certain difficulties. Introducing at one half of depth L a layer of porous material, it will be possible to damp down anti-resonant vibrations and thus to smooth out the drops in curve α.

The obtaining of $\alpha > 0.9$ is altogether feasible within the interval of one octave. Assuming $\alpha_1 = 0.9$ with boundary frequencies of 100 and 200 Hz, we obtain, according to (5) and (6), $L = 38.4$ and $\frac{\Sigma}{K} = 38.4$. The choice of values $\sum$ or K can be made arbitrarily. The curve for α is given in Fig. 3, *I*. Taking L and $\frac{\Sigma}{K}$ three times less, we shall shift the curve of absorption by an interval 3:1 upwards; this case corresponds to curve I' in Fig. 3. Calculation of α, according to formulae (1), (2), (3), is not quite accurate; in taking into account the length of pipe L according to formula (7), instead of curve *I* a dotted curve for α is obtained with the same structural dimensions, close to it up to 200 Hz.

With given $\frac{\Sigma}{K}$ the outlay of porous material (in orifices σ) per unit

wall area is proportional to the square of aperture diameter D^2, and therefore it is expedient for the sake of economy of material to make the apertures as small as possible. Friction per unit area of σ will be proportional to D and, accordingly, in using small apertures we may take porous material with a finer mesh (i. e. a wire-gauze).

For the absorber, which gives $\alpha > 0.9$ within the range of one octave, we shall obtain, according to (5), $L = \frac{\lambda_1}{8.8}$; at 100 Hz we obtain $L = 38.4$ cm. If we extend the absorption range with the help of an additional internal layer with friction up to the interval 8 : 1 (3 octaves), then $L = \frac{\lambda_1 \frac{\lambda_1}{8}}{4.4} \approx \frac{\lambda_1}{5}$. At 100 Hz we obtain $L = 68$ cm. The obtaining of high absorption in a wide frequency range therefore requires a thick layer.

The friction coefficient (averaged in volume) will (with $L = 38.4$ cm) be equal to: $r = \frac{R' \rho c}{L} = \frac{1.22 \cdot 43}{38.4} = 1.3 \frac{\text{mech. Ohms}}{\text{cm}^3}$. We take here into account only the main friction in aperture σ, since the additional friction will be very small. Cremer ([3]) has computed that in order to obtain $\alpha > 0.9$ above 100 Hz it is necessary to use a layer of very friable absorbing material (loose cotton wool) with a friction coefficient $r = 1.3 \frac{\text{mech. Ohms}}{\text{cm}^3}$ and a thickness $L > 64$ cm. The fact of the existence of such close values for two so widely different types of absorbers deserves our attention.

A monoresonance absorber can be very effectively designed, provided not too high absorption is aimed at. For instance, taking $\alpha > 0.5$ within the range of 60 — 600 Hz with a maximum $\alpha = 0.64$ (Curve *II* in Fig. 3), we obtain $L = 23.4$ cm, $\frac{\Sigma}{K} = 38$. Such an absorber vill be very useful for reducing of low-frequency noises in a number of cases (ventilating ducts, chambers for testing aeroplane motors). Curves *III* and *IV* in Fig. 3, obtained by calculation in the range up to 2000 Hz (the data of the respective systems are shown in the figure), recall the appearance of absorption curves for material of the type of «Nashkote», «Sanacoustic», «Celotex», etc.

For constructing resonance systems it is necessary to known how to realize such a value of friction R, which would stand in conformity with the requirements of the design. The investigation effected in our laboratory by S. T. Ter-Osipiantz has shown that friction in apertures closed with a porous material (textile fabric, wire-gauze) hardly depends upon frequency and that friction per 1 cm^2 for different kinds of nets lies within the range from $0.1 \frac{\text{mech. Ohms}}{\text{cm}^2}$ (mesh of gauze about 0.1 cm) to $8 - 10 \frac{\text{mech. Ohms}}{\text{cm}^2}$ (the thickest wire-net of reps plaiting). Textile fabrics give yet greater magnitudes of friction. By means of superimposing various nets and textile fabrics, it is easy to obtain the desired magnitudes of friction in the apertures.

Comparison of calculated values of absorption coefficient show a coincidence (with precision of 5%) with the results of measurements by the method of standing waves in a tube (16 cm diameter). Coincidence of maximum values of α was obtained (reaching in a number of instances, as was precalculated, up to unity), as well as the points of drops in the anti-resonance regions.

In resonance absorption systems porous material giving friction can be made use of with a by far greater efficiency than usually. For

instance, for a system with depth of $L=25$ cm, $\sum=1.8^2$ cm^2, and apertures $D=0.3$ cm in diameter, by introduction into the meshes of a flimsy gauze with friction of 0.9 $\frac{\text{mech. Ohms}}{\text{cm}^2}$, we obtain at a frequency of about 200 Hz a maximum of absortpion reaching as high as a unity, as well as a number of secondary maximums. If an entire layer of the same gauze is placed at the same distance of 25 cm from the wall, this will give, as shown by computation, an absorption of maximum 0.08. In order to raise the absorption at the maximum up to the former value, we have to take a friction of 42 $\frac{\text{mech. Ohms}}{\text{cm}^2}$ which can be accomplished by superimposing 47 layers of the same material. Computing the expenditure of gauze in both cases, we find that to obtain the same absorption, the resonance construction, owing to the highly effective application of porous material, permits of taking a 2160 times (!!) lesser quantity of material. The great importance of this fact was pointed out by the author as early as in 1936 ([5]).

Under the conditions of reverberating rooms, it will always be possible to cover sufficiently large surfaces with simple monoresonance absorbers of two different types, doing this in such a way, as to make the region of the maximum of one system overlap the drop in the curve of the other (low frequency) system (cf. for instance the Curves I and I' in Fig. 3). Both systems together can easily give a considerable mean absorption coefficients ($\alpha > 0.6$) within a wide range of frequencies.

II. Systems with more than One Layer. For a double-layers (biresonance) system the input impedance in the aperture may be determined on the basis of the solution of Lagrange's equation for a system with two degrees of freedom, and it will be found equal to:

$$Z = Z_1 + \frac{E_{12}^2}{\omega^2 Z_2}, \qquad (8)$$

where

$$\left.\begin{aligned} Z_1 = R_1 + j\left(M_1\omega - \frac{E_{11}}{\omega}\right); \quad Z_2 = R_2 + j\left(M_2\omega - \frac{E_{22}}{\omega}\right); \\ M_1 = \frac{\rho\sigma_1^2}{K_1}; \quad M_2 = \frac{\rho\sigma_2^2}{K_2}; \\ E_{11} = \frac{\rho c^2\sigma_1^2}{V_1}; \; E_{22} = \rho c^2\sigma_1^2\left(\frac{1}{V_1} + \frac{1}{V_2}\right); \; E_{12} = \frac{\rho c^2\sigma_1\sigma_2}{V_1} \end{aligned}\right\} \qquad (9)$$

(notations of the dimensions of the system see in Fig. 1).

Z_1 and Z_2 are the impedances of two partial systems obtained under the condition of closing apertures σ_1 or σ_2; M_1 and M_2 are added masses in apertures σ_1 and σ_2; E_{11} and E_{22} are the elasticities of two partial systems, and E_{12} is the elasticity of the coupling.

With $\omega_{22} = \sqrt{\frac{E_{22}}{M_2}}$, $Z = R_2$, and if R_2 is small, then value Z can become very large and α small (a case of anti-resonance); making R_2 sufficiently large, we are able to smooth out the drops in curve α. It will be easy to show that with low frequencies the double system works as a monosystem with volume $V_1 + V_2$, as if the perforated screen between volumes V_1 and V_2 was absent. A screen with aperture σ_2 does not, in this way, produce any change in the region of low frequencies and permits of making use of the full length $(L_1 + L_2)$ of the whole volume for obtaining a great absorption. At the same time in the anti-resonance region (ω_{22}) by means of sufficiently great friction R_2 we can prevent drop in the curve of α. At high frequencies, α (8) is determined mainly by impedance Z_1 of the first resonator, and by its rational designing it

is possible to provide high values of α. These tentative considerations indicate that the first system is to be taken less deep than the second one ($L_1 < L_2$).

Rational design for a double system is more complicated. If we let $\alpha > \alpha_1$, at both boundaries $f_2 : f_1$ we obtain but two equations, while we have to determine seven parameters: R_1, σ_1, L_1, R_2, σ_2, L_2 and $\sum$.

It will be easily seen from (8) that on the vector diagram in Fig. 2 impedance Z for a double-layer system gives a loop in the region near the axis of the abscissae. The object of design consists in making this loop get wholly inside a circle corresponding to $\alpha = \alpha_1$. As evidenced by the detailed investigation performed by V. S. Nesterov in our laboratory, it is possible with the help of biresonance system to obtain coefficients of absorption $\alpha > 0.9$ within the range from 100 to 500 Hz (2.5 octaves) with a depth of layer of about 50 cm. The absorption curve will be usually found to be double-humped with hollow in the middle and a steep drop beyond the boundary frequencies. The checking by the tube method gave fair agreement with theoretical design. Absorption of the order of $0.5-0.6$ may be obtained within a very wide range, which affords the possibility to meet the greatest variety of requirements in respect of desired characteristics, as required by architectural acoustics.

The investigation of double-layer system by tube method has shown a good confirmation of theory. For a three-layer absorber it will be easy to find the input impedance by resorting to the same method:

$$Z = Z_1 + \frac{E_{12}^2}{\omega^2\left(Z_2 + \frac{E_{23}^2}{\omega^2 Z_3}\right)}. \qquad (10)$$

This formula is constructed in analogy to (8), only instead of Z_2 it comprises the quantity $Z_2 + \frac{E_{23}^2}{\omega^2 Z_3}$, constructed in the same way as (8) and giving the input impedance in the second aperture σ_2. A rational calculation according to this formula is yet considerably more complicated than according to formula (8). However, the tentative computations have shown that a three-layer system with a total depth of the order of 60 cm can yield an absorption of $\alpha > 0.9$ within the range from 100 to 2000 Hz (4.5 octaves). An absorption of $\alpha > 0.9$ within the range from 100 to 4000 Hz is undoubtedly obtainable with a four-layer system having a thickness of $50-60$ cm.

The calculation for a polyresonance absorber performed by G. D. Maliuğinetz [6] has exhibited the full possibility of obtaining $\alpha > 0.9$ within a range of frequencies from 100 to 5000 Hz. Such an absorption system should, according to tentative computation, consist of 22 perforated sheets and it ought to have a thickness of about 1 m; it is certain that this system can be made in a simpler way and thinner.

Calculation and tests of resonance sound-absorbers have been carried out by the author with the participation of a group of co-workers. The work had for ist purpose to find a high absorber for the ceiling of the big hall of the Palace of the Soviets.

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Received
8. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PHYSICS

LUMINESCENT PROBE IN A GAS DISCHARGE

By V. FABRIKANT

(Communicated by S. I. Wawilow, Member of the Academy, 9.II. 1939)

The intensity of radiation emitted by the discharge ([1]) is strongly influenced by the nature of radiation diffusion in the body of the discharge. Yet, from measurements of the intensity of the radiation emitted by the discharge we are able to draw nothing but very indirect conclusions as to the radiation diffusion.

As a matter of fact, when making such measurements, the physicist feels about the same as an astronomer who would study spectra emitted by stars and nebulae and try to base on such observations some conclusion as to the regularities of distribution of radiant energy in the depths of those luminaries. This is particularly true of resonance lines characterized by strong absorption.

1. The object of the present communication is to describe a new method which permits of a direct study of radiation diffusion in the body of gas.

This method consists in the use of luminescent probes introduced in the gas volume*. A luminescent probe transforms the energy of incident radiation into a radiation with a sharply altered wave length; this radiation ceases to be absorbed by the gas, and freely escapes from the latter. One may judge of the characteristics of the radiation diffusing through the gas by the brightness of the probe. The latter is measured either by means of a spectral device, or by the stroboscopic method, in order to isolate the emission of the discharge itself. In the case of stroboscopic observations, the discharge tube is being fed with alternating current, the probe is covered with an inertia luminophorus, the readings being taken at the dark moments of the discharge (through a stroboscopic disc).

By varying the type of luminophorus and the mode of observation, one succeeds in studying individual spectral lines in the discharge.

2. Without making any assumptions as to the nature of radiation diffusion in gas, one has to take into account some simple relations between the brightness of a luminescent probe and the characteristics of the radiation. The brightness of the probe is proportional to the energetic illumination created on the surface of the probe by the radiation which excites the luminescence. There may be some deviations connected, say, with saturation of luminescence; these can, however, be avoided by an adequate selection of the luminophorus.

* A brief description of this method has been reported at the Session of the Physical Group of the Academy of Sciences of the USSR (Jan. 27—28, 1938) ([2]).

The average brightness of a spherical probe is proportional to the voluminal density of radiation. The alteration of brightness of a flat probe turned by 180° is proportional to the projection of Pointing's vector on the direction of the normal to the plane of the probe. The difference between the brightnesses of these two positions of the probe must be, therefore, proportional to the cosinus of the angle of turning of the normal with regard to a certain direction in the space.

Analogous relations are observed in common light measurements ([3]). They may be used to find the value and direction of Pointing's vector at any point of the volume of the gas.

3. In the case of great optical densities, when the analogy between photon and atom diffusion may be applied([1]), the characteristics of a luminescent probe may be given an interesting and simple interpretation. From the analogy with atoms, the number of photons falling within a unit area of a flat probe will be expressed by the following equations ([4]):

$$N_{\downarrow} = \frac{N\bar{c}}{4} + \frac{\bar{c}\lambda}{6}\frac{dN}{dz}, \quad N_{\uparrow} = \frac{N\bar{c}}{4} - \frac{\bar{c}\lambda}{6}\frac{dN}{dz}, \tag{1}$$

where $N_{\downarrow}$ and $N_{\uparrow}$ denote the two sides of a flat probe, N is the photon concentration, $\bar{c}$—the average velocity of photons $\left(\frac{\lambda}{\tau}\right)$, λ—the length of the mean free path of photons $\left(\frac{1}{\bar{k}}\right)$([1]), z being directed normally to the probe surface.

As $B_{\downarrow}$ and $B_{\uparrow}$, the brightnesses of the respective sides of the probe, will be proportional to $N_{\downarrow}$ and $N_{\uparrow}$, one may readily obtain the following equations:

$$B_{\downarrow} + B_{\uparrow} = \text{const}\, N\bar{c}; \tag{2}$$

$$B_{\downarrow} - B_{\uparrow} = \text{const}\, \frac{dN}{dz}\bar{c}\lambda; \tag{3}$$

$$B_{\downarrow} - B_{\uparrow} = \text{const}\, \frac{d}{dz}(B_{\downarrow} + B_{\uparrow}); \tag{4}$$

$$\lambda = \frac{3}{2}\,\frac{B_{\downarrow} - B_{\uparrow}}{B_{\downarrow} + B_{\uparrow}} : \frac{d\ln(B_{\downarrow} + B_{\uparrow})}{dz}. \tag{5}$$

An experimental checking of the validity of equation (4) will make it possible to determine the limits of the analogy between the diffusion of atoms and that of photons.

Equation (5) permits of a direct determination of the effective length of the mean free path of photons in gas. Let us recall that the non-exponential character of absorption makes the regularities of the photon diffusion in gas ([5]) most complicate, since the exponential law of atom diffusion in gas is of prime importance when developing equation (1). The experimental checking of (4) and (5) is, therefore, of interest on principle ground.

Curiously enough, the complex formula obtained by Zemansky ([6]), as may be seen from the comparison of data tabulated, practically leads, on the basis of Kenty's calculations, to a simple proportionality between the equivalent optical density $\bar{k}l$ and the square root from the optical density for the core of the line $k_0 l$ (Fig. 1).

We have already taken advantage of this fact in a paper cited above ([1]).

4. We shall give here some of the preliminary results of the application of luminescent probe to the study of the discharge. Experimental data will be exposed in more detail in a paper by Butaeva.

The measurements were carried out in a mercury lamp at a vapour pressure of about 10^{-3} mm Hg, with a current rating 3 A and a tube of 28 mm in diameter. A probe (2×10 mm) was transferred with the help of magnet along the diameter of the discharge, the surface of the probe being normal

to the diameter. The probe was coated with villemite, having a sharp maximum of luminescence yield in the region of 2500 Å. Under the discharge conditions investigated, such a probe does practically react only to a resonance line of 2537 Å.

A thick curve B in Fig. 2 denotes the brightness of the probe as a function of the position of the probe. The direction of observation is from right to left. It is noteworthy that the brightness of the probe has a maximum not on the axis of the discharge. One may easily see that the positions of the probe symmetrical to the axis of discharge correspond to the brightnesses $B_{\downarrow}$ and $B_{\uparrow}$ respectively. Thus, the asymmetry observed testifies to the important rôle played in equation (1) by the gradient of photon concentration.

Making good use of equation (5), one may estimate the length of the mean free path of the photons. Under our experimental conditions, it was found to be of the order of a few mm, which is in good agreement with the values determined from elementary constants and the shape of the lines (5). The curves $\frac{S}{2}$ and Δ in Fig. 2 represent the semi-sum of $B_{\downarrow}$ and $B_{\uparrow}$ and their difference respectively. The first curve shows the run of the volumi-nal density of radiation in a cross-section through the discharge, under definite conditions only, while the second one must accurately coin-cide with the run of the value of the radial flux of radiation in the discharge.

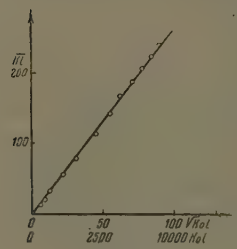

Fig. 1.

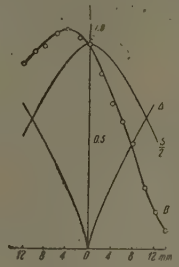

Fig. 2.

5. As well as in the case of an electrical probe, the question as to the disturbing action of a luminescent probe upon the discharge assumes great importance.

Theoretically this problem may be reduced to the solution of diffusional problems, the boundary conditions being put on the surface of the probe.

Experimentally the problem is solved by comparing the results of measurements carried out by means of probes of different sizes.

Let us but mention here that the disturbing action of the probe is at its maximum at the verge, when the discharge happens to be squeezed between the wall and the probe. Owing to this fact, the extreme right point of the experimental curve fails to be quite reliable.

On the other hand, the gradient of density in the region close to the discharge axis does but slightly differ from nought. Therefore, in order to estimate λ one should have recourse to the values of B referring to about the medium of the radius.

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Received
9. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PHYSICS

ON THE HYSTERESIS OF AN ALTERNATING CURRENT ELECTRIC DISCHARGE

By B. L. GRANOVSKY and E. V. MERZLOUKHOVA

(Communicated by S. I. Wawilow, Member of the Academy, 9.II.1939)

Physical investigations of electrical discharges in gases are usually conducted with direct current, while theories of gas discharges are evolved almost solely for a steady state. In the application of our general knowledge on discharges to a discharge of alternating or pulsating current, the assumption is ordinarily made that this discharge will be quasi-stationary at low frequency (e. g. 50 Hz), i. e. that all its characteristic quantities are at any moment of time determined by the instantaneous magnitude of the discharge current.

There exist, however, quite a number of facts which testify to the presence of non-equilibrium phenomena in a low frequency discharge. Under this head in first place fall a certain difference in the static and dynamic volt-ampere characteristics of a discharge ([1],[2]), and the consequent phase displacement between tension and current in the discharge ([3],[4]), which we may characterize as the hysteresis of the volt-ampere characteristic. Further, to the same class of phenomena will also belong the dependence of the resistance of a discharge upon frequency ([4]), the dependence of the length of triation upon the derivative $\frac{di}{dt}$ ([5]), as well as many other phenomena. If we exclude phenomena brought about by changes in the density or composition of the gas due to electrophoresis or heat developed in the discharge, we shall find that the principal factor disturbing the equilibrium of a discharge is the finite rate of accumulation and disappearance of ions. Particularly significant is the rate of deionization which is a slower process. This slow rate should, in particular, lead to the condition that the concentration of ions in a discharge and the resultant densities of random currents of ions and electrons should be determined not only by the magnitude of the discharge current, but also by changes in the latter with time. It is this surmise that has been checked by us in the light of experimental evidence.

With the said object in view we carried out a number of experiments in a mercury arc (with a liquid cathode), at a frequency of 50 Hz. The type of discharge tube, as well as the method used for control of pressure, have been described elsewhere in a number of papers from our Institute. For detailed description of the set-up see the work of one of us. With the help of an electromagnetic oscillograph oscillograms were taken of the discharge current and simultaneously of the ion current to a cylindrical probe placed along the axis of the discharge (d=0.5mm, l=12 mm, U_s=—30V).

On each oscillogram, besides discharge and probe currents, there were

also to be found: 1) calibration lines for discharge current i_a and probe current i_s, for which purpose the circuits of both elements were switched over from the discharge tube to a d. c. circuit; 2) a control photograph, for which both elements were connected in parallel and made to record the same quantity. The last photograph permitted to make a check of the coincidence of the initial positions of both traces as well as of the elements, and—in case of need—to estimate the magnitude of the respective corrections.

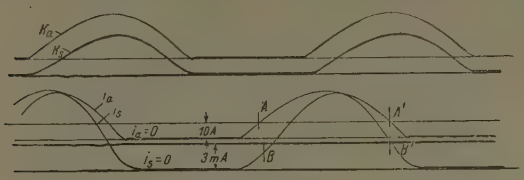

Fig. 1.

An example of such an oscillogram is to be found in Fig. 1; the conditions of discharge are here as follows: tube $d=65$ mm, pressure of mercury vapours (reduced to 0°C) 36μ, average current—10 A. We see that different values of the ion current (points B and B') correspond to equal values of the discharge current (points A and A' respectively). Consequently, the value of the ion current is not determined by the magnitude of the discharge current alone; with increase of i_a, the values of $i_s=i'_s$ are less than with the decrease of i_a (i''_s). For a quantitative investigation of this phenomenon let us represent the data of this oscillogram, by plotting i_s against i_a (Fig. 2, the second curve from above). We may characterize the result as demonstrating the existence of a hysteresis of ion current in the discharge. If the quantity $\delta=(i''_s-i'_s):\frac{i''_s+i'_s}{2}$ is to be taken as a quantitative measure of this hysteresis, we shall find that the hysteresis is in the given experiment positive and increases towards the end of the arcing period. This result is qualitatively easily accounted for by making the assumption that the ion current characterizes in a certain measure the density of ions in the discharge. We are then indeed entitled to interpret the observed phenomenon as a consequence of the finite rate of ionization and deionization of the vapour. It is natural that δ should be higher where $\frac{di}{dt}$ is large, i. e. in a region far from the current maximum—a fact which is observable in the present experiment and, generally, in all the cases examined, when the density of the discharge current is moderate.

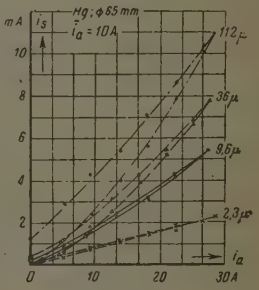

Fig. 2.

Our interpretation of the phenomenon observed makes us conclude that the slower the deionization proceeds, i. e. the higher is the pressure and the greater is the volume of the deionized space, the more pronounced will be the effect. Both these circumstances have been fully confirmed in our experiments. In Fig. 2 are to be found the result obtained, while using one and the same tube ($d=65$ mm) at different pressures: 2.3, 9.6, 32 and 112μ. We see that with increasing pressure the hysteresis of the ion current increases not only in absolute, but also in relative magnitude.

Thus, with $i_a=10$ A we find that at low pressures $\delta\approx 0.1$; at 32 μ $\delta=0.3$; at 112 μ $\delta=0.45$. Increase of the diameter of the tube acts in a similar manner. Fig. 3 represents the tabulated results of the oscillograms taken in tubes with $d=65$ and 105 mm at equal pressure $p^*=65$ μ. In the tube with $d=105$ mm the

hysteresis of the ion current is appreciably greater than in the tube with d =65 mm. Both the mechanisms involved in this case bring strong evidence to speak in favour of the interpretation of the discovered phenomenon, as a consequence of the finite rate of accumulation and disappearance of ions.

In the discussion of this result we should have in mind the following considerations: the magnitude of the ion current flowing to the axial probe is determined, in the first place, by the concentration of ions at discharge axis n_0. The power of the discharge current $i_a = e\,\bar{u}_e\,N_e$, where e is unit charge; $N_e = 2\pi\int_0^R n\,(r)\,r\,dr$—number of electrons per 1 cm discharge length, $n(r)$—concentration of electrons at distance r from the axis, $\bar{u}_e$—mean axial velocity of the electrons in the discharge. Further, $\bar{u}_e = bX$, where X is longitudinal potential gradient, b—mobility of electrons, usually slightly dependent on X. In its turn, X is connected with the total voltage drop in the tube U_a. It follows, therefore, from the well-khown shift of U_a relatively to i_a that the velocity of electron drift u_e at the first period of arcing ought to exceed the second. Consequently, there should also exist a time shift between i_a and N_e*. Since X is in advance of i_a, N_e lags behind i_a. Further, in an unstabilized discharge, N_e (number of ions or electrons per 1 cm of discharge length) should not necessarily be proportional to the concentration of ions at the axis n_0. At low pressure the recombination of ions proceeds almost solely at the walls, whereas ionization takes place throughout the volume.

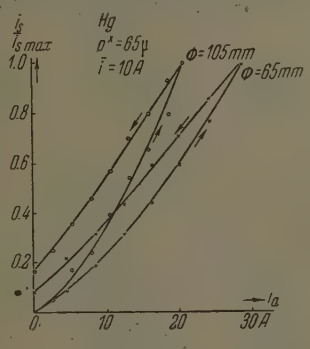

Fig. 3.

Cases are, therefore, possible when, with increasing discharge current, the transverse gradient of concentration will be less than with its reduction; in other words, with $\frac{di_a}{dt} > 0$ the curve of the distribution of concentration over the section is more gently sloping, while it will be steeper with $\frac{di}{dt} < 0$. In this case in the first half of the arcing period the ratio $\frac{n_0}{N_e}$ will be smaller than in the second half. This circumstance will have its effect upon the shift of n_0 relative to the discharge current, this effect working in the same direction as a change in the drift velocity of electrons.

Thus, two different circumstances make possible the existence of a multi-valued relationship between the concentration of ions at the discharge axis and the magnitude of the discharge current. Both of them find their ultimate explanation in the finite rate of deionization. Their joint result, as displayed in the above described phenomenon, offers a characteristic example of an unstabilized electrical discharge of low frequency.

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9. II. 1939.

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* It is impossible, strictly speaking, to determine it as phase displacement, since the curves are non-sinusoidal and the difference in phase between them is not constant.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PHYSICS

X-RAY EMISSION FROM THE ISOMERS OF RADIOACTIVE BROMINE

By L. I. ROUSSINOW and A. A. YUSEPHOVICH

(Communicated by A. I. Joffe, Member of the Academy, 1. II. 1939)

It has been shown recently that the radioactive transformations of the isomeric bromine nuclei are accompanied by a soft radiation with an energy of $\sim$30 000 eV (1). This radiation has been supposed to consist of inner conversion electrons resulting from the transition of the metastable Br nucleus to the ground state. The present research was undertaken with the view of checking the above hypothesis.

If the soft radiation is really due to conversion electrons, the disintegration of the radioactive bromine should be accompanied by the emission of characteristic X-rays. When, during conversion, an electron escapes from the atomic shell, a rearrangement of the latter will result, and the subsequent emission of X-rays.

The following procedure was adopted for the experimental detection of the X-rays. Radioactive bromine was prepared by exposing ethyl bromide to slow neutrons, the active bromine being extracted by chemical methods. The target was placed between the poles (12×6 cm^2 surface area) of an electromagnet, the intensity of the field attaining 6000 Gauss. The radiation was measured by means of a Geiger-Müller counter with 10 μ thick aluminium walls filled with Ar at a pressure of 400 mm, a voltage of 3000 V being applied to the counter.

With the magnetic field on, no electrons resulting from the disintegration of the radioactive bromine should enter the counter, since their maximum energy is 2 MeV, while the distance between counter and target was 3 cm. However, under actual conditions, a great number of impulses could be observed in the counter. A few preliminary experiments sufficed to show that, in addition to the γ-rays accompanying the β-disintegration of the bromine and carrying an energy of several hundreds of ekV, the radioactive target emitted an electromagnetic radiation with low energy which could be completely absorbed by 1 mm Pb.

Further experiments showed that the intensity of the X-ray emission decayed with a 4 hour period. The X-rays are therefore emitted by the radioactive Br_{II} nucleus undergoing conversion.

An absorption method was used to determine the wave length of the above electromagnetic radiation. Absorption by aluminium yielded a wave length of about 1 Å which corresponds to the K_α-line of the X-ray spectrum of bromine.

In order to increase the accuracy of the wave length determination, the method of selective absorbers was used.

The X-ray spectrum of bromine is known to contain two strong lines: the K_{α_2}-line ($\lambda = 1041.60$ XE) and the K_{α_1}-line ($\lambda = 1037.59$ XE). A marked change in the absorption coefficient of the K-radiation must be observed, when arsenic absorbers are replaced by selenium absorbers. Indeed, the K-absorption limit of Se is 977.73 XE, while that of As is 1042.63 XE, so that in the latter, contrary to Se, photoelectric absorption of X-rays in the K-level will be observed. The magnitude of the change of the mass absorption coefficient, as calculated from Jönssen's formula and from Siegbahn's [2] data, is 6.9. The value of μ/ρ for the K-radiation of bromine is 199.0 in As, and 28.1 in Se.

The absorbers were prepared from chemically pure As_2O_5 and SeO_2; some sheets of filter paper were impregnated with a solution of the given compound, the absorber was dried in a vacuum, and the amount of absorbing substance was determined by weighing.

As may be seen from Fig. 1, in As and in Se absorbers, given the same mass per unit surface, the X-rays emitted by radioactive bromine are absorbed to a different extent. The experimental values coincide with the calculated values of the absorption coefficients.

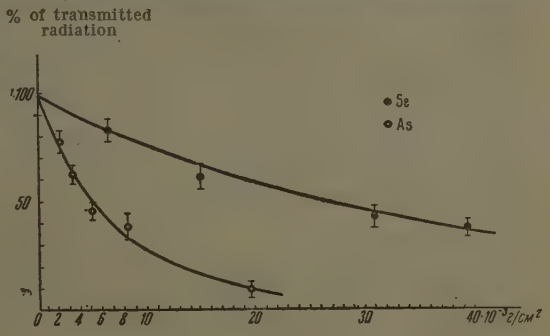

Fig. 1.

In As the absorption of the rays is 5 times greater than in Se. The observed change in the absorption coefficient is somewhat less than might be expected from calculation, but this may be due to the fact that, with the given experimental arrangement, secondary X-rays arising in the As absorber could not be completely eliminated. The influence of this secondary emission was shown by special experiments.

The number of quanta of the observed X-rays constitutes 50% of the number of disintegration electrons emitted by the excited Br nucleus. The number of disintegration electrons was deduced from counts with no magnetic field applied. The number of quanta of the X-ray emission was derived from the counting rate and the sensitivity of the argon counter. The results obtained are in fairly good agreement with the data on the fluorescence yield of the K-level of bromine, namely 56%. The remainder of the ionized atoms emit Auger electrons.

The X-rays observed cannot be ascribed to a capture of the K-electron by the Br nucleus, since, in this case, the X-rays would be emitted by the Se atoms. For selenium, the wave length K_{α_2} is 1106.52 XE and K_{α_1}—1102.48 XE. These wave lengths cannot produce the fluorescence of the As atoms, and their absorption in As and Se must give similar curves.

On the basis of the above results, an attempt was made to develop a scheme for the transformation of isomeric Br nuclei.

In principle, no objection can be made to the view that the metastable Br nucleus first passes to the ground state by inner conversion, after which β-disintegration sets in. Another scheme is equally possible: the metastable Br nucleus might first emit β-rays, transforming into krypton, after which

the excited Kr nucleus passes to the ground state. In this case it would be the Kr nucleus which undergoes inner conversion. The absoption measurements in As and Se do not permit to decide between the two schemes, since the lines K_{α_2} Kr =982 . 1 XE and K_{α_1} Kr =978 . 1 XE also lie within the limits of absorption of Se and As. We therefore measured the absorption of the X-rays in mercury and lead. The absorption limit L_{III} has a wave length of 1007.5 XE in Hg and of 949. 2 XE in Pb.

While the X-rays from bromine are unable to excite the fluorescence of the Pb and Hg atoms, the krypton rays can produce fluorescence in Hg. For the K-line of bromine, calculation yields values of μ/ρ equal to 82.3 in Hg and 86.5 in Pb, while μ/ρ for the K-line Kr is equal to 74.3 in Pb and 254 in Hg. Thus, should the conversion take place in the krypton, the mass absorption coefficient should decrease 3.4 times upon replacing Pb by Hg.

The study of the absorption of the X-ray emission in Hg and Pb was conducted in the same manner as that in As and Se. The absorbers were prepared from chemically pure $Pb(NO_3)_2$ and $HgNO_3$.

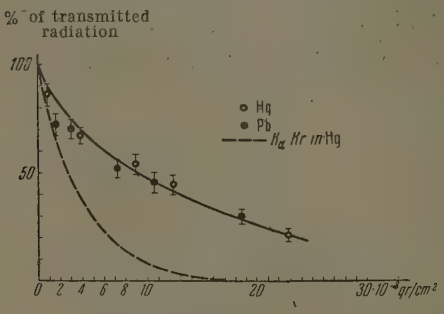

Fig. 2.

The results are shown in Fig. 2. It is easily seen that the absorption in Pb is the same as in Hg, the experimental coefficients of absorption coinciding with those obtained by calculation. It is clear that we have to deal with the first of the two schemes, namely the conversion of the Br nucleus during nuclear isomerism.

The present study can thus be regarded as affording an experimental proof that the transition of the metastable nucleus to the ground state can proceed by way of inner conversion.

The results leave little doubt as to the scheme of this transition. The metastable Br nucleus first passes to the ground state by inner conversion, after which the β-disintegration sets in.

We intend next to proceed to a more detailed quantitative investigation of the problem, in order to examine the experimental facts in connexion with the theory of nuclear isomerism.

In conclusion we wish to thank Prof. I. V. Kourtschatow for valuable advice.

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Received
5. II.1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PHYSICS

NEUTRON SCATTERING IN NORMAL AND PARAHYDROGEN

By A. LEIPUNSKI and D. TIMOSHUK

(Communicated by S. I. Wawilow, Member of the Academy, 15. II. 1939)

The scattering cross-sections of gaseous para- and orthohydrogen for neutrons with the energies 0.037 eV and 0.019 eV were measured. The cross-sections of orthohydrogen were found to range from $55 \cdot 10^{-24}$ cm^2 to $71 \cdot 10^{-24}$ cm^2, the cross-sections of parahydrogen from $47 \cdot 10^{-24}$ cm^2 to $22 \cdot 10^{-24}$ cm^2. In accordance with Brickwedde, Dunning and others the existence of a spin interaction between the neutron and proton and the virtual character of the singlet level of deuterium were established.

Teller and later Teller and Schwinger ([1]) pointed out that the existence or non-existence of spin interaction between neutron and proton could be ascertained by measuring the scattering of slow neutrons in para- and orthohydrogen.

In the case when the velocity of the neutrons is sufficiently high, the molecular state of hydrogen will in no way affect the scattering of neutrons by protons, since the neutron wave length will be too small compared to the distance between the protons in the H_2 molecule to give rise to any interference phenomena. The neutrons will be scattered by the protons in the same way, as if the latter belonged to any other compound with the same binding energy. In the case of sufficiently slow neutrons, the wave length of which is comparable to the distance between the protons in the H_2 molecule, the relative orientation of the spins, if any, should exert appreciable influence upon the scattering of neutrons by the H_2 molecule.

For ortho H_2, in which spins are parallel, the scattering (in the limit case when the neutron velocity is near zero) will correspond to an interaction with a double spin. For para H_2, whose spins are anti-parallel, the spin interaction in the limit case will practically be absent. Teller and Schwinger have also shown that the character of the scattering will be essentially dependent on the sign of the singlet level of the deuteron. Thus, if the level is real, i. e. the energy of the deuteron in singlet state is less than the energy of the separate proton and neutron, the cross-section of scattering by parahydrogen must increase by 30% when passing from neutrons with the energy 0.037 eV (equivalent to the thermal equilibrium of neutrons and protons in paraffin at room temperature) to neutrons with the energy 0.012 eV (equivalent to the liquid air temperature). If the level of the deuteron is virtual, the ratio of the cross-sections for the same neutron velocities must be 100 : 1. This holds for elastic scattering only.

Thus, the study of the scattering of slow neutrons by para- and orthohydrogen affords information concerning not only the forces involved in the

neutron-proton interaction, but also the sign of the singlet level of the deuteron.

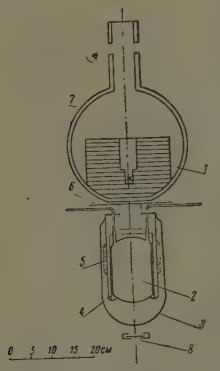
Apparatus. The study of neutron scattering by para- and orthohydrogen was carried out with an equipment consisting of:

a) a device for production of a mixture enriched with parahydrogen,

b) a scattering apparatus adapted for a gaseous scattering substance.

The device for the production of parahydrogen consisted in a steel tube filled with charcoal and immersed in a Dewar vessel containing liquid hydrogen. Gaseous hydrogen was passing from the bomb into a thin spiral copper tubing surrounding the coal containing tube, where it was liquified and ran down into the lower part of the tube with charcoal. Hydrogen was kept over the charcoal until the parahydrogen content rose as high as 80 or 90%. The enriched mixture was supplied to the scattering apparatus through a lead tube.

The scattering apparatus shown in Fig. 1 is a chrome-nickel steel bomb *2* with rounded bottoms and walls 2.5 mm thick. The bomb forms part of a metal Dewar vessel *3*. To the upper part of the bomb are soldered two lead tubes, by one of which the bomb is connected to the parahydrogen apparatus and by the other to the manometer, analyzer and gas-holder. The inside walls of the bomb are coated with a thin layer of shellac to avoid contact between parahydrogen and iron. This procedure permitted to keep constant the composition of the mixture throughout the experiment. The bomb was surrounded with a cadmium filter *4* 0.6 mm thick. The interstice between the bomb and the inside wall of the Dewar vessel was filled with liquid nitrogen, the level of which was 15 mm below the upper end of the bomb. The level was controlled by a hypsometer. In the lower part of the Dewar vessel was a heater (not shown in the figure) which prevented moisture to accumulate on the wall of the Dewar vessel facing the ionization chamber. The source of slow neutrons was a metal Dewar vessel *7*, inside of which was placed a paraffin block with a source of neutrons about 200 mg of radium mixed with beryllium. In the paraffin were imbedded iron disks to increase the heat conductivity. The lower part of the Dewar vessel was provided with thin nickel windows for letting out the neutrons. The efficiency of the apparatus in slowing down the neutrons has been investigated by Leipunski and Roussinow(²). The ratio of the absorption coefficient of neutrons in boron with the Dewar vessel filled with liquid nitrogen to the absorption coefficient of the neutrons slowed down by the paraffin at room temperature is 1.53, which corresponds to an effective «temperature» of the neutron of 123°K. The average energy of the neutrons is then 0.019 eV. The detector used was an ionization chamber with boron *8* connected to a linear amplifier. The impulses were recorded by a thyratron counter. The effective solid angle subtended by the chamber at the scatterer was 0.045 steradian. The composition of the mixture was analyzed by measuring the thermal conductivity.



Measurements and Results. The measurements of the scattering were carried out as follows: the number of counts without cadmium filter over the bomb and without hydrogen in the bomb was measured during 10 minutes. Then the aperture in the cadmium filter was screened with a cadmium sheet, and the number of counts was measured again. The scattering bomb was then filled with para- or orthohydrogen to a certain pressure, the hydrogen was cooled down to nitrogen temperature during approximately 3 minutes, after

which the same measurements were repeated as without hydrogen. After approximately 30 minutes the mixture was analyzed and let off to the gasholder. The measurements were carried out with neutrons of two energy distributions: 1) in equilibrium with protons in the paraffin at room temperature, 2) the same at liquid nitrogen temperature. The results are tabulated in the table.

Neutron energy	Cross-section $\times 10^{-24}$ cm^2	Composition of the mixture	Cross-sections $\times 10^{-24}$ cm^2 extrapolated for pure components of the mixture
300°K	53	75% of ortho 25% of para	for ortho 15
129°K	59	Do	» » 71
300°K	48	15% of ortho 85% of para	for para 47
129°K	30	Do	» » 22

Note. The cross-section values tabulated are corrected for the geometrica dimensions of the apparatus and for secondary scattering.

Extrapolating the obtained values of the cross-section for the pure components of the mixture, we obtain the following figures:

for neutrons with the energy equivalent to 123°K:
ortho $\sigma_o = 71 \cdot 10^{-24}$ cm^2, para $\sigma_p = 22 \cdot 10^{-24}$ cm^2,
and for neutrons with the energy equivalent to 300°K:
ortho $\sigma_o = 55 \cdot 10^{-24}$ cm^2, para $\sigma_p = 47 \cdot 10^{-24}$ cm^2.

These data are in qualitative agreement with the theoretical predictions for the virtual level of the deuterium and agree with the conclusions of Brickwedde, Dunning and others [3]. The fact that the parahydrogen cross-section for neutrons at 123°K exceeds that predicted by theory seems to be due entirely to the participation in the scattering of neutrons with energies higher than the thermal energies lying within the absorption limit of cadmium, and of neutrons which are in thermal equilibrium with the protons in the paraffin, but whose energy is above 0.024 eV. These faster neutrons are liable to be scattered inelastically and therefore to cause transformation of parahydrogen into orthohydrogen which requires 0.024 eV. The cross-sections for the gaseous scatterer are in better agreement with theoretical predictions than the data of Brickwedde and others: the increase of the orthohydrogen cross-section when passing from room temperature neutrons to neutrons of liquid nitrogen temperature is small, whereas Brickwedde and his co-workers report a nearly twofold increase.

Thus, the comparison of the experimental results with the theory of Teller and Schwinger points to the existence of spin dependent forces between neutrons and protons, and to that the singlet level of the deuteron is virtual, without permitting to make any conclusions as to the radius of action of the nuclear forces, because of the inelastic collisions which occur when measuring the scattering cross-section of parahydrogen.

We are indebted to Eng. V. Dementi for his help in carrying out these experiments.

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Received
5. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
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PHYSICAL CHEMISTRY

TIME THRESHOLDS OF MITOGENETIC EFFECT ON COAGULATION AS RELATED TO CONCENTRATION OF COLLOID AND ELECTROLYTE COAGULATOR

By M. A. VLADIMIRSKAJA

(Communicated by L. I. Mandelstam, Member of the Academy, 2. II. 1939)

To find how the time threshold value of the mitogenetic effect is related to the concentrations of the colloid and the electrolyte during coagulation processes is of theoretical interest in solving the problem, as to whether the mitogenetic radiation, which is observed during the coagulation of the colloid, is a result of the very process of particle aggregation (1), or there may be a secondary radiation in the solution during coagulation. The purpose of this paper is to ascertain the parts played by both factors, viz. the concentration of the colloid and that of the electrolyte in coagulation processes.

According to the theory of mitogenetic effect, the fundamental decisive factor in problems of obtaining the mitogenetic effect is the «regime of the mitogenetic source» after Gurvich.

Molecular dispersion sources, which give a statistically even quant flux, are the least favourable for obtaining the induction effect (the «levelling factor» predominates). On the other hand, coarse dispersion radiation is favourable for obtaining the induction effect (the «excitation effect» predominates).

If during coagulation the detector responds to secondary radiation, produced in the dispersion medium, it is absolutely immaterial what changes take place in the dispersed phase. If, however, the latter is the radiation source, the nature of the radiation depends on the number of foci formed during aggregation in unit volume and unit time. In this case changes in the concentration of the colloid and in the electrolyte may tell upon the effect. The coagulation of a colloidal solution through the addition of KCl was taken as a process producing radiation. Yeast agar cultures served as detectors. The method was as follows: the colloid and the electrolyte were poured successively in various volumes into a chamber with a quartz glass. An agar block was placed opposite the quartz glass. Time was counted off by a stop watch from the moment of adding the KCl solution. After adding KCl the colloid was stirred with a glass rod. We used two methods of discharging: the continuous and the interrupted one (by inserting a disc with sectoral cuts every 30°; 1500 rotations per minute). The initial concentrations were as follows: 0.07 g colloid in 100 cm^3 solution and 1 *n* KCl. In the subsequent experiments the colloid was diluted 4, 16, 64 and 128 times.

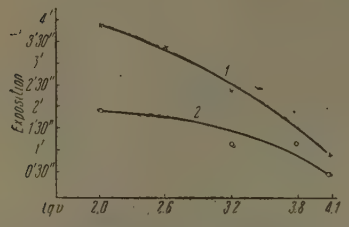
Table 1

Effect of Concentration of the Colloid V_2O_5 on the Time Threshold During Coagulation Processes (electrolyte KCl)

Exposure	Concentration of colloid									
	0.07%		0.018%		0.004%		0.001%		0.0005%	
	with fract.	without fract.	with fract.	without fract.	with fract.	without fract.	with fract.	without fract.	with fract.	without fract.
0′15″									+14 — 8 — 5	
0′30″							— 9 +13		+25 +37 +55 +30	— 6 — 2
0′45″					+ 6 — 9 —17		± 0 — 3 —15		+30 +37	+ 4 —14
1′0,″			+ 9 + 4 + 1		+ 9 +14 +38 +39	—13 + 9	± 0 +15 +29 +36	+ 8 — 9	+43 +36	+24 +20 +52 +23
1′15″			± 0 + 2 + 3 — 4		+48 +47 — 5		+34 +18			
1′30″	+10 — 2		+21 +22 +15		+35 +21 +46 +28	+12 +23 + 4 ± 0	+24 +20 +19	+ 4 — 8 — 6		
2′0″	+27 —18 +20 +41	+11 —11	+28 +29 +20		+37 +50 +27	+17 + 7 +27 +34		+39 +35 +21		
2′30″	+22 +20 +25		+45 +36	+22 — 2 + 4		+35 +63				
3′0″	+32 +27	+ 7 + 8	+30 +54	+19 + 4 +11 + 9						
3′30″		+ 8 —13 +18		+22 +37						
4′00″		+23 +42 +45		+39 +35	Note. The figures show the effect in per cent.					

Table 1 shows sharply the relation between the time thresholds and the concentration of the colloid: the time threshold decreases with the concentration. When the concentration is 0.07%, we get an effect after two minutes,

exposure, while when the colloid is diluted 128 times the effect is observed after 30 seconds' exposure. The results are represented graphically in the figure. If the supply is fractionated, the time threshold decreases all along the curve.



Time threshold as influenced by the mode of stimulus supply. *1*—without fractioning, *2*—fractioned.

As we have seen, the phenomena described here may be accounted for by the properties of the detector: in diluting the colloid we seem rarefy the quant flux and create the best conditions, under which the field of the detector can be utilized the most fully.

The same method was applied in studying the result of diluting the electrolyte. The colloid was taken always in a concentration of 0.018%, while the normal solution diluted 4, 16 and 64 times successively. In this connexion we were successful in establishing that the more the electrolyte is diluted, the more the time threshold increases. The explanation of this phenomenon should be sought in the gelatination properties of dilute solutions of electrolytes when added to colloids.

Table 2

Effect of Concentration of Electrolyte KCl on the Change of the Time Threshold During Coagulation of V_2O_5

Exposure	Concentration of electrolyte		
	Normal solution	4/*n* solution	16/*n* solution
1'30''	+21 +22 +15	— 2 + 3 ± 0	+ 3 — 2
2'	+29 +29 +20	+28 +46 +26 +11	± 0 — 2
3'		+23 +25 +33	—15 +35 +29

The capability of the particles of colloidal solutions to form jellies instead of precipitates when aggregated is very great (2) and extends even to systems, which contain no more than 0.01 % of dry substance. This capability is doubtlessly connected with the well-defined anisodiametricity of the particles, whose localized coagulation results in the formation of the space and not the compact type of structure (3).

It will be obvious that the formation of a space network of interlaced fibres, for instance, makes it impossible for the same particles to aggregate further. The process goes on with a great time decrement (as a result of the fixation mentioned) and the conditions are unfavourable for the supply of stimulus. The lower the concentration of the electrolyte coagulator, the more closely

the process of aggregation approaches typical jellification. Hence, as it seems to us, the increase in the time threshold with concentration decrease.

The conclusion to be drawn from our data, in accordance with the principles of the mitogenetic effect set forth above, is that radiation is connected immediately with coagulation of the dispersed phase, not with the substance (the dispersion medium). The effects described would not appear, if the effect had originated from secondary radiation.

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Received
31. XII. 1938.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

CHEMISTRY

ON THE BORON CONTENT OF SALT DEPOSITS IN THE SOUTH BASIN OF THE PERMIAN SEA AND IN SALT LAKES OF THE CASPIAN LOWLANDS

By I. N. LEPESHKOV

(Communicated by N. S. Kurnakov, Member of the Academy, 26.I.1939)

In 1934 extensive borate deposits were discovered in the Permian layers of Western Kazakhstan (Inder lake region) (1). This gave us the possibility to discontinue importing borate raw materials. The growing demand for boron in this country puts us before the problem of the need of forced searches for borate deposits in other regions which were covered by the Permian Sea.

Table 1

Salt layer	№ of well	Depth at which the test was taken, in m	B_2O_3 (in weight %)
Ashche-Bulak	1	166.5— 167.5	0.0143
»	1	266	0.0205
»	1	329 — 334.9	0.0081
»	1	354 — 358	0.0059
»	1	427 — 430.5	0.0055
»	1	609 — 611	0.0027
Kairovka	5 (1405)	449 — 454	0.019
»	5 (1405)	634	0.0070
»	5 (1405)	671 — 677	0.0115
»	3 (1403)	1087 —1094	0.0095
»	3 (1403)	1103 —1118	0.0076
»	3 (1403)	1141 —1147	0.0080
»	3 (1403)	1156 —1162	0.0069
»	3 (1403)	1162 —1169	0.0242
Krasnoyarka	1	221 — 225	0.0234
»	1	228 — 238.2	0.0096
»	1	300.6— 306	0.0098
»	1	372	0.0049
»	2	390 — 400	0.0044
»	2	488 — 531	0.0167
Ishimbayevo	801	681 — 692	0.0117
»	801	692 — 702.9	0.0132
»	801	702 — 712	0.0061
»	801	712 — 723	0.0146
Ozinky	2 (a)	581 — 590	0.0174

During the last few years the Institute of General and Inorganic Chemistry has been carrying on an investigation of the potassium content of the salt layers of Western Kazakhstan and neighbouring regions. Work has also been done in studying the genesis of Inder borates and the possibility of finding them on the extensive territory of the Caspian lowlands (2).

In this paper data are given on the boron content in salts and bore-hole waters of some of the potassium beds of Chkalovsky, Aktyubinsky and Saratovsky regions and Bashkir ASSR, and also in the brines of a number of salt lakes of Western Kazakhstan.

Table 1 shows the boron content in the salt cores from the following beds: Ashche-Bulak (Aktyubinsky region), Kairovka, Krasnoyarka (Chkalovsky region), Ozinky (Saratovsky region) and Ishimbayevo (Bashkir ASSR).

A high content of boron (up to 0.02% B_2O_3) was found in all the salt samples studied (the B_2O_3 content in the water of the Azov Sea is about 0.00056%; $d^{24}/_{24}=1.0049$) (3). Bore-hole waters, taken from various depths of the Yar-Bishkadak, Kirov Indusry, Buranchino and other wells (Bashkir ASSR), also contain a large quantity of boron (Table 2). In some of the tests

Table 2

Place where the test of the bore-hole water was taken	№ of well	Depth in m	d 18/18	Total of all salts	B_2O_3 (in weight %)
Yar-Bishkadak	805	950	1.1283	17.22	0.0082
» »	10	900	1.2018	27.90	0.0580
» »	810	850	1.2235	27.95	0.0627
» »	9	700	1.2032	26.41	0.0033
Kirov Industries	215	880	—	12.50	0.0164
» »	188	930	1.2013	24.65	0.0317
Buranchino Industries	4	850	—	4.19	0.0029
» »	607	300	1.2149	23.77	0.0563

the B_2O_3 content reaches 0.06%. It is quite significant that brines from the lakes of Western Kazakhstan are rich not only in potassium, but also in boron (Table 3). This high boron content in the brines of some salt lakes of Western Kazakhstan was noted also by I. B. Feigelson and A. A. Kozhevnikova (4). This shows that the salt lake brines were mineralized at the expense of the

Table 3

Salt lakes from which tests were taken	d 25/25	Weight per cents	
		Total of all salts	B_2O_3
S. Karabatan	1.250	30.73	0.024
North Iskine	1.280	31.28	0.075
Ural-Bai-Sor	1.208	26.33	0.046
Aral-Sor	1.280	31.35	0.015
Chartakty	1.242	29.30	0.022
Ak-Kyz	1.285	31.82	0.05
Inder	1.2265	27.108	0.185
Inder (from the wap. basin)	1.2965	32.90	0.480
Inder (from the wap. basin)	1.4007	37.39	0.80

deep Permian salt deposits. The connexion between the surface salt lakes and the deep salt deposits is especially manifest in Inder lake. At a concentration of 24.5 Be' the B_2O_3 content in Inder lake brines goes up to 0.135% (the potassium content in the lake's brine is about 2%).

A large quantity of boron was found in the brines of North Iskine lake (B_2O_3=0.075%). It should be noted that the potassium content in Iskine lake brines equals 0.7%.

The salt deposits of the south basin of the Permian sea are characterized by a high content of potassium, sulphuric acid, boron and bromine.

The high boron content in the salt deposits and bore-hole waters of the beds mentioned above and in the salt lakes of Western Kazakhstan shows that borate deposits may possibly be found not only in the Inder lake region, but also in other regions of Western Kazakhstan and neighbouring districts. Thus, in 1934 a few beds of hydroboracite were found among the Iletsk salt deposits ([5]). A few years ago we spoke of the potassium content of the salt deposits of the Caspian lowlands according to only several analyses of salt lake brines (Inder, Iskine), in which large amounts of potassium were found. But even then N. S. Kurnakov put forward the possibility of finding potassium deposits in the salt domes of the Caspian lowlands.

At present as a result of investigations, carried out by the Institute of General and Inorganic Chemistry of the Academy of Sciences, and of oil boring («Embaneft», «Vostokneft», «Aktubeneft» etc.), we have been successful in bringing to light extensive deposits of potassium-magnesium and other salts ([6]) in the regions mentioned above.

Potassium and other salts have been observed here in the form of polygalite ($K_2SO_4 \cdot MgSO_4 \cdot 2CaSO_4 \cdot 2H_2O$), kainite ($KCl \cdot MgSO_4 \cdot 3H_2O$), kieserite ($MgSO_4 \cdot H_2O$), glauberite ($Na_2SO_4 \cdot CaSO_4$), bishophite ($MgCl_2 \cdot 6H_2O$), carnallite ($KCl \cdot MgCl_2 \cdot 6H_2O$) etc. All these salts except carnallite were discovered for the first time in deposits of the Union.

At the present time it may be asserted that the Soviet Union has a new potassium base. Unlike the Solikamsk potassium deposits, we have here not only chlorides, but also sulphates of potassium which make better potassium fertilizer.

Along with investigations for potassium, other are also needed to expose boron deposits in Western Kazakhstan and neighbouring districts.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GEOCHEMISTRY

AN INVESTIGATION INTO THE VISCOSITY OF METEORITES

By M. P. VOLAROVICH and A. A. LEONTIEVA

(Communicated by W. I. Vernadsky, Member of the Academy, 11.II.1939)

In connexion with our foregoing works on the investigation of the viscosity of molten glasses (1), slags (2) and rocks (3), it appeared to us to be of interest to perform measurements of the viscosity of meteorites with a view to elucidate certain peculiarities of their texture, as well as the conditions of their formation.

From among the meteorites obtained from the Meteorite Collection of the Academy of Sciences of the USSR, we, while working in the laboratory of the Petrographical Department of the Institute for Geological Sciences, have studied the following specimens: tectites (4) from Indo-China, moldavites (5) and a stony meteorite «Saratov» (fall in 1918).

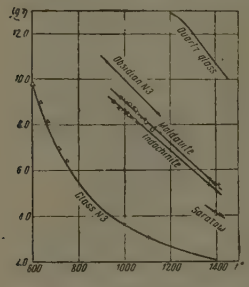
Before proceeding to measurements of viscosity, preliminary determinations were effected of the softening point of the meteorites. For indochinite this was found to lie at about 1070°, while for moldavite at 1100°. The softening point for the latter proved to be much lower than the temperature indicated by Suess (5) who believed that at 1250° moldavites did not yet begin to soften. After holding the tectites at a temperature of 1400° for an hour, they caked together into a homogeneous mass without bubbles, this mass next sintering into a glass of the same colour as the initial specimens had.

For measurements of the viscosity of tectites within their softening range we resorted in the main to the same method of the extension of a rod under load which had formerly been made use for investigations of quartz glass (6), as well as of obsidians (7). A prism of $3.0\times0.6\times0.6$ cm in size was cut out of the tectites, which was next ground down in its middle over a length of 0.6—0.7 cm in such a way, as to make the section equal to 0.3×0.3 cm. By means of a special clamp the prism was then located vertically in a vertical tubular electric furnace. Then to its lower end, with the help of a cathetometer, weight P was suspended, whereupon the determination was effected of the velocity of extension v of the specimen at constant temperature under the action of a fixed weight. The evaluation of viscosity in absolute units—in poises—was accomplished according to the formula (8): $\eta=\frac{Pl}{Sv}$, where l is the length of the middle narrow portion of the specimen in cm, while S is cross-sectional area in cm^2. In calculations due account was taken of the fact that during the experiment length l was liable to increase gradually, while the cross-section tended to diminish.

For investigating the viscosity of meteorites in molten state we applied Stokes'method of a falling of ball, which we had formerly used formeasu-

rements of the viscosity of boric anhydride and the system SiO_2—B_2O_3. This method was for the first time made use of at high temperatures by B. V. Derjaguin ([9]).

The meteorites were melted in a corundum crucible with a capacity of 20 cm^3 and were then held for an hour at 1400°, in order to secure the removal of bubbles. Then a platinum ball of 2—2.5 mm in diameter was dropped into the melt; upon the expiration of a period of 30—40 minutes the crucible was rapidly taken out of the furnace and quenched. By breaking the crucible it was possible to determine the distance which the ball had traversed in its fall, to calculate the rate of its fall, and then according to Stokes' formula to calculate the viscosity. The density of the meteorites was determined at room temperature and it was found to be: for indochinite—2.309 g/cm^3, for moldavite—2.387 g/cm^3, and for the meteorite «Saratov»—1.516 g/cm^3 (for the latter upon remelting); for the introduction of these data into Stokes' formula, an approximate correction was made on the basis of data on the density of other silicate melts at elevated temperatures ([10]). Thus, at $t = 1400°$, $d = 2.054$ cm^3 was taken for indochinite, $d = 2.1$ g/cm^3 for moldavite and $d = 1.35$ g/cm^3 for the meteorite «Saratov».



For the meteorite «Saratov» the viscosity of the siliceous portion was measured, for which purpose it was finely crushed and freed from metallic iron by means of a strong magnet. This portion of the meteorite began fusing only at about 1320°.

The results obtained are graphically represented in the figure where temperature is plotted along the axis of the abscissae, while the logarithm of viscosity is plotted along the axis of the ordinates; data obtained according to the rod extension method are shown by circles, while dots show the results of repeat measurements. Crosses denote the data obtained according to the method of the falling ball. As will be seen, it was found possible, by dint of interpolation, to connect the points obtained by two different methods for tectites. The connexion is made by means of a common curve, whose shape comes close to that of a straight line; the graphs having an altogether lawful course.

Measurements for indochinite embrace the range of temperatures from 950 to 1400°, the viscosity being respectively equal to $1.06 \cdot 10^9$ and $1.22 \cdot 10^5$ poises; the range of temperatures for moldavite constitutes 980—1410°, while the respective viscosity equals $1.7 \cdot 10^9$ and $2.3 \cdot 10^5$ poises. We succeeded in carrying out measurements for the meteorite «Saratov» only within the range of temperatures between 1390—1410°, the viscosity being equal to $1.48 \cdot 10^4$ and $1.15 \cdot 10^4$.

In the figure are to be found temperature curves illustrating the relationship between quartz glass and obsidian № 3, according to our previous measurements ([6,7]), plotted here for comparison. The curve for ordinary glass is drawn in conformity with Procter and Douglas'([11]) data, which coincide with the results of measurements made by M. Volarovich and D. Tolstoi ([12]), these latter effected by the method of rotating cylinder. The data on the viscosity of the same glass obtained by us by the method of rod-extension are plotted in the diagram as triangles, while the measurements accomplished according to Stokes' method are represented by crosses. As evidenced by the diagram, the accuracy of our method is sufficiently satisfactory.

From the figure it is seen that the curves for the meteorites occupy an intermediate position between those for quartz and ordinary glass. This is explained by their chemical composition. The viscosity of molten silicates is in the

first place brought about by the content of SiO_2 in them, wherefore quartz glass with a content of silicic acid equal to 99.9% was found to be 100 000 times more viscous than tectites containing about 70—80% SiO_2. The part taken by other oxides is, in the main, as follows: Al_2O_3 enhances the viscosity, while alkali, contrariwise, lower it. On account of this, ordinary glass containing about 20% of Na_2O appears to be 10 000 times less viscous than tectite, though the content of SiO_2 in it is also about 72%. At the same time obsidian shows a viscosity which is somewhat higher than that of tectites, since it contains 2% more Al_2O_3. The viscosity of the meteorite «Saratov» is very high, though the latter contains but 44% of SiO_2, this being due to the fact that a very large amount of MgO is available in it.

In comparing the viscosity data of meteorites with the results available for various rocks, it was found that meteorites fail to square with the mechanism, previously established by one of the writers and L. I. Korchemkin ([13]) for the dependence of the viscosity of rocks upon their acidity coefficient α according to F. J. Loewinson-Lessing. This fact should be ascribed to the cosmic origin of meteorites. The high viscosity of tectites also speaks against their terrestrial origin, the viscosity being indeed so great as to prevent all the entrapped bubbles of gas from coming out in course of the fusion process. We believe, however, that only experiments on the synthesis of meteorites under conditions of tremendous temperatures and pressures, which could be accomplished, say, in a set-up of the Ramsauer type ([14]), could definitely settle the question of the origin of tectites.

Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

HYDROCHEMISTRY

HYDROCHEMICAL CHARACTERISTICS OF THE WATERS OF THE NEVA BAY

By I. V. BARANOV

(Communicated by W. I. Vernadsky, Member of the Academy, 9.II.1939)

The history of hydrochemical investigation of the Neva bay and the gulf of Finland is closely connected with their practical utilization. In this respect we must note in the first place the work of Skorikov ([8]), who took part (1905—1906) in the complex expedition to the Ladoga sea, organized by the Municipal executive committee with the aim to improve the water supply of Petersburg. The work of this expedition included short investigations of the Ladoga sea, the Neva, the Neva bay, as well as the eastern part of the gulf of Finland. From this hydrochemical material Skorikov came to the following conclusion: «The Neva bay is filled with clean fresh water, the composition of which is very near to that of the Neva river, as shown by the analysis of my water sample carried out by Lebedinzev».

In 1911—1912 the Neva bay was subjected to an hydrochemical survey on account of a sheme of a water supply from the Ladoga sea to Petersburg. This work, which lasted 15 months, was carried out by Zaleskij ([5]), Wulf, Dubjanskij, Przhibitek, Spindler and others. The problem put before them was to clear up the influence of sewage waters of the Neva on the hydrochemical regime of the Neva bay and to find the most suitable place for the discharge of sewage. These authors also shared the opinion of Skorikov that the area between Petersburg and Kotlin island is filled with fresh water during the winter. From 1920 till the end 1938 the Neva bay and the eastern part of the gulf of Finland were investigated by several organizations under the direction of late Prof. Derjugin.

In 1935 the economic organizations were confronted with the problem of the influence of sea and sewage waters on the Neva bay and of adequate discharge of the latter; this problem became most urgent especially because of the development of industrial establishments, sanatoriums etc.

Taking into account this problem, as well as several others, we conducted during the period 1935—1937 an extensive investigation of this basin under the direction of Prof. K. M. Derjugin and J. K. Novodranov. The investigations were usually carried out at morning hours, beginning with the first station. The first station was situated in the Morskoj Kanal (Marine Channel) in front of the Peterhof Biological Institute building, while the 2nd station was at 2 km off shore. For the sake of uniformity, both stations were oriented on local objects always in the same way. Now, after

an almost three year period of regular investigations we have at our disposal rather abundant material, partly made use of in this preliminary report.

In winter and at the beginning of summer the water level of the Neva bay is considerably lower than in the late summer and autumn. Shokalsky and other hydrologists explain this autumnal rise, observed regularly every year, by the predominance at this time of winds of the northern rhumbs, driving water into the gulf of Finland; in winter the influence of the winds decreases, owing to the ice-cover.

Our investigations showed the variation in salinity of the Neva bay waters to be mainly due to pulsation of the so-called inner waves. Inner waves had been found in the Danish sounds by Pettersson ([9]) in 1909 and at the head of the gulf of Finland by Prof. Derjugin ([2],[3],[4]) in 1924. Owing to this phenomenon, the tension of the sea water in the Neva bay is subject to fluctuation on a large scale at the same point. Thus, 7. III. 1936 the chlorine content of the bottom layer of the Marine Channel was but 9 mg per litre, it reached 10. III—920.8, 13.III—1313.0, 16.III—1133.0, 18. III—863.4 and 21. III—325.5 mg/l respectively. 30.IV there was 8.1, 3. V—1651.68; 6.V—1444.32, 9. V—2033. 21 and 12.V—1832.40 mg/l at the same point. Seasonal observations have shown that the saltness of the Neva bay differs at winds of ost and south-west rhumbs, being the higher, the more continuous are winds of ost rhumbs. At south-west winds a brack water current penetrates through the south gates and reaches the pier of the Marine Channel.

The oxygen regime of the Neva bay is also at a considerable degree dependent on the variations of salinity. In the absence of sea water the bottom curve of O_2, as a rule, coincides closely with the surface curve, whose salinity during the summer period approaches 100% saturation. In winter O_2 deficiency attains 30—35% in the southern region. One of the causes of this deficiency is the absence of interchange of fresh and salt water through the south gate, so that in winter a belt of stagnant Neva water is formed along the Peterhof coast. Furthermore, there remains here a considerable part of town refuses. In summer the decrease of oxygen in the bottom layer is the greater, the higher the salinities. Thus, for instance, 15. II. 1936 we have 99.9% O_2 at the depth of 0 m and 77.3% (Cl'—2896.7 mg/l) at the bottom; 20. VIII. 1937 we have: 96.8% O_2 at the depth of 0 m and 70.9% (Cl'—3245 mg/l) at the bottom. These conditions are in full agreement with the oxygen regime of the Baltic sea. As known, owing to a two-tier distribution of waters in this sea, its bottom layers contain an insignificant amount of O_2.

The seasonal variation of oxygen is closely connected with the dynamics of free carbon dioxide, the concentration of HCO_3' and the active reaction. The minimum of CO_2 content corresponds, as a rule, to a maximum of O_2. Considerable variations of pH are due to the fact that the Neva bay waters have but a feeble capacity (mean HCO_3 concentration=27—30 mg/l).

The active reaction of the waters of the south region during summer is slightly alkaline (pH=7. 2—9. 2). In winter the decrease of pH at the second station is greater than at the first one. Sometimes surpassing of the neutral point (6. 89) can be observed. A comparison of the HCO_3' values with the salinity shows that when the channel is filled with fresh water, the vertical stratification of HCO_3' is but slightly expressed as well as at the second station.

The sea water produces an increase of total alkalinity. Under such conditions the amount of fixed carbon dioxide in the bottom layer increases up to 75 mg/l. The maximum amount of free CO_2 (9.37 mg/l) at the second station was observed on the 3.III. 1936, the difference between the bottom and surface layers equalling 0.49 mg/l. After the breaking up of the ice the total amount of carbon dioxide decreases as compared to the winter period. Thus, previous to breaking up on the 3.IV there was 5.27 mg/l of free CO_2

at the bottom, 6. V—4. 31 and 30. V—2. 37 mg/l. A total disappearance of free CO_2 was observed in 1936 during the period of 15—21 June. It fell within the period of intensive growth of phytoplankton. It is to be noted that, as well as in 1936, the absence of free CO_2 in the surface layer of the second station in 1935 was observed only for a short period. Thus, for instance, no free CO_2 was formed on the surface the 15. VI, whereas the 18. VI its amount was as high as 2.21 mg/l.

The Neva bay waters are not particularly rich in organic matter. Thus, the hydrochemical survey of 11. IX. 1937 showed the oxidability of the whole basin to vary at that day from 5.91 to 7.77 mg O_2 per litre. Let us give a brief account of the data yielded by stationary observations.

In summer 1935, the maximum oxidability (11.12 mg O_2 per litre) in the surface layer of the second station was observed the 27. VIII, while the minimum (6. 47 mg O_2 per litre) was recorded the 15. VII. The vertical stratification was, as a rule, but slightly expressed. Thus, it reached 2. 41 mg O_2 per litre only the 15. VII, fluctuating in the other five cases from 0.35 to 1.43 mg O_2 per litre. The table shows the mean value of oxidability at the first and the second stations in 1935—36.

Mean Values of Water Oxidability

	South region of the Neva bay (2nd stat.)	Marine Channel (1st stat.)
Summer 1935	surface — 8.88 mg O_2 per litre	7.36 mg O_2 per litre
	bottom — 9.86 » » » »	8.82 » » » »
Winter 1936	surface — 10.11 » » » »	7.88 » » » »
	bottom — 10.49 » » » »	8.27 » » » »
Summer 1936	surface — 9.27 » » » »	8.92 » » » »
	bottom — 10.21 » » » »	9.76 » » » »

This table permits of drawing the following conclusions: first, it is evident that the inshore part of the south region is characterized by a higher oxidability, than the Marine Channel both in summer and in winter; secondly, tha rate of oxidation is less intense under the ice-cover than in summer, which results in an accumulation of organic matter in the Neva bay, especially in its southern part. This accumulation is, indoubtedly, enhanced by the stop of water exchange through the Oranienbaum bank; thirdly ,the comparison of year-round observations of 1935—36—37 with those of 1911—12 shows that during this period of 24—26 years the Neva bay waters assumed a higher oxidability. This phenomenon is, probably , due to the discharge of Leningrad sewage which were more contaminated, than in 1911—1912.

The analysis of the above data and their comparison with the pH and O_2 values, as well as with the system of currents, point to the obvious inexpedience of the discharge of sewage waters into the southern region of the Neva bay. In winters (throughout the period of ice-cover) the whole area is characterized by an extremely slow water exchange which creates a situation insecure with regard to stagnation of organic refuses. In summer the sewage waters can be driven by the so-called inshore Peterhof current to Leningrad instead of the gulf of Finland. The discharge of the sewage waters into the Marine Channel is also connected with danger, since along its bottom a pulsating brackish current is running in a direction opposite to that of the surface waters.

Summarizing all the before said, we can reach the following conclusions:

1. Owing to a continuous system of currents in the Neva bay, all the principal hydrochemical elements (HCO_3', free CO_2, SiO_2, oxidability, $Fe^{\cdots}$, O_2 etc.) show no pronounced vertical stratification under normal conditions.

2. The greatest variation of the salt content of the Neva bay waters is shown by the chlorine, the amount of the latter ranging between 3 and 3200

mg per litre, i. e. different regions of the Neva bay can be filled either with fresh water from the Neva river, or with waters from the gulf of Finland having a low salinity. In conformity with this fact, the vertical stratification of salinity is almost nil in the first case and very distinctly pronounced in the second. . .

3. The brackish currents enter the bay from the gulf of Finland along the bottom of the Marine Channel and over the Oranienbaum bank.

4. In the case of winds of ost rhumbs, the salinity of the Neva bay is increased by the waters of the Marine Channel, while south-west winds act in the same way driving waters over the Oranienbaum bank.

5. In winter the whole southern part of the Neva bay is entirely filled with fresh water from the Neva river. In summer under the action of south-west and ost winds, the salinity in the bottom layer reaches sometimes 4.04‰.

6. Under normal conditions the waters of the bay are characterized by an insignificant total mineralization: HCO_3'=27—30, the total hardness 1.24° (German), CaO—9—9.6, Cl—3—5, Fe··· about 0.04 mg per litre, i. e. are but slightly buffered. Owing to this fact:

7. The reaction of the waters is not constant. The pH value in the surface layers varies during the year from 6.9 to 9.2, the ($H\cdot$) in the bottom layer of the Marine Channel waters being the lesser, the higher the salinities. A more or less noticeable vertical stratification of pH can be observed only in the case of a total absence of free CO_2.

8. In summer, the Neva bay reveals for a long period an insignificant deficiency of O_2. In the bottom layers this deficiency of O_2 rises with the increase of salinity. An insignificant excess of O_2 is usually observed in July and August in the inshore waters of the southern region. This fact can be accounted for by intensified photosynthetic processes of phytoplankton. In winter the reserve of O_2 is more intensively consumed in the southern region of the Neva bay.

9. The values of free CO_2, CO_3'', O_2 and pH in the summer period show a circular horizontal distribution with a gradual increase of the three elements from the pelagic part to the coast. In winter the accumulations of free CO_2 and HCO_3' are more intense in the southern region of the bay, as compared to the Marine Channel. The vertical stratification of these elements in winter is less pronounced than in summer. The annual variation of HCO_3' in the Neva bay waters depends rather on the fluctuations of salinity connected with the pulsation of sea water than on biological factors.

10. The comparison of the results of the year-round observations on the changes of the principal hydrochemical elements in 1935—37 with those of 1911—12 shows that the waters of the Neva bay in the last years became more contaminated.

11. The variations in the content of silicic acid in the Neva bay is brought about chiefly by biological factors. In winter its amount and oxidability are greater in the coastal parts, than in the Marine Channel.

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Received
10. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

MINERALOGY

NEW DATA ON THE MINERALOGY OF THE ILMEN MOUNTAINS

By A. F. SOSEDKO

(Communicated by A. F. Fersman, Member of the Academy, 10.II.1939)

Up to the present it has been held that in the Ilmen mountains granite pegmatites and syenite and nephelite pegmatites are localized separately. The former are encountered only at Mount Kossaya, while the latter occupy the main Ilmen ridge. As an exception only five outcrops of nephelite pegmatites have been reported from Mount Kossaya among the outcrops of pegmatites in the granite magma. The investigations of 1937, resulting in the discovery of five new pegmatite nepheline veins at Mount Kossaya, showed that the dimensional correlations of different pegmatites are of much more complicated character.

New outcrops of nephelite pegmatites at Mount Kossaya are found in the following areas.

1. Westward from the Blumov mine № 50. Between the Blumov mine and the swamp separating Mount Kossaya from the main Ilmen ridge to the left from the road leading to the Blumov mine small hills (eltysh) are rising. At their ridge among granitogneisses the first of the newly found nephelite veins is passing. It begins 50 m north-west from the Blumov mine and proceeds in a latitudinal direction almost up to the swamp over a distance of more than 250 m. The vein is thin, rarely up to 1 m in thickness, usually 0.5—0.4 m, its course being interrupted by breaks and pinches. It is composed of albite, nephelite, black mica, occasionally pink mica, sodalite, aeschynite, zircon, ilmenite; in one thin-section pyrochlore has been found.

2. Another outcrop of nephelite syenite has been found 25—30 m to the north from the mine № 96 with nephelite syenite. Only fragments of nephelite syenite in the eluvium have been found.

3. A third outcrop of nephelite-bearing pegmatites was found near the Cryolite mine (№ 69). A nephelite vein is exposed at a small hill (eltysh) lying approximately 80 m north-north-west from the Cryolite mine. This vein occurs 10 m eastward from the Amazonite mine № 96. The vein follows a north-eastern direction and has been traced over 20 m. It consists of albite, biotite, nephelite, cancrinite, sodalite and occasionally of zircon and aeschynite. In the south-western portion, in the northern marginal contact pink feldspar with pink lithium-bearing mica has been encountered.

4. A fourth outcrop has been detected 100 m north from the third one. By accumulations of rock fragments the pegmatite vein has been traced westward approximately over an area of about 150 m. Its composition is similar to the preceding.

5. Finally, south-west from the Cryolite mine at the middle between this and the Corundum mine there has been found a block with nephelite syenite.

These findings point to the fact that the Ilmen mountains are as yet insufficiently studied. Even in the region of such well-known mines, as the Cryolite and the Blum mines, new veins may be found whose mineralogical composition is quite new for their surroundings. Of particular interest are the findings of nephelite veins in the vicinity of the Cryolite mine. In this connexion interest is roused again to the genesis of cryolite, which, wherever found, in Greenland or in the Ilmen mountains, is enclosed in pegmatites occurring near nephelite syenites.

New data have been attained as to mineralogy of nephelite and syenite pegmatites. Attention has been turned to the fact that the ilmenites from Mount Lokhmataya, mine 16^{II}, when crushed, gave powders of different colouring, from light-grey to black. This circumstance suggests that this ilmenite is heterogeneous, and consists at least of two minerals, a different content of which in the specimen produced a different colouring of the powder. In order to find out whether it was so, a number of experiments and investigations was carried out. By means of a powerful electromagnetic separator the «ilmenite» has been separated into two fractions: the non-electromagnetic of a greyish-green colour and the electromagnetic of a black colour. The specimens of ilmenite, when polished, especially on etching the surfaces with concentrated hydrochloric or fluoric acid, were found to be composed of two minerals, one of a steel-grey colour, the other with a reddish tinge. The first showed no etching effect, while the second solves and blackens.

A solubility test showed that the steel-grey mineral remained almost unsolved after 5 days of solution in concentrated hydrochloric acid, while the second passed rather soon into solution. The solubility was controlled by testing the solvent on Ti. A streak-test (on a porcelain plate) showed that the steel-grey mineral gave in the reflected light a pale greyish-green streak, while the streak produced by the other one proved deep black.

By means of a blowpipe the difference between these minerals is easily established. Thus, the mineral giving a greyish-green streak gives a negative reaction with Mn, while the mineral with the black streak gives a bead peculiar to Mn.

The behaviour of both minerals is different as regards weathering. On the exposed surface of the walls of fissures, one may observe that the «ilmenite» is in some cases covered with a greenish film, while in others this film is absent. This coating was found to be inherent only to the mineral with the black streak. Thus, from the weathering products it is possible to disclose the inner structure of the specimen and the correlation of these two minerals.

In order to establish the nature of these two minerals, a chemical analysis has been effected. The mineral with the black streak proved to be ilmenite. As to the mineral with the greyish-green streak, it was found to approach ilmenorutile. Its chemical composition corresponds to the formula $FeO.NC_2O_5.16.5\ TiO_2$. From its physical properties it closely resembles the ilmenorutile from the Lobachev mines.

At Mount Lokhmataya, in the 16^{II} mine ilmenorutile is encountered in rounded shapes, up to 20 cm in diameter. A study of some hand specimens of «ilmenite» in the reflected light showed that the ilmenorutile occurs there in an intimate intergrowth with ilmenite. Frequently ilmenorutile forms large individuals, 3—4 of which compose a specimen. They are encountered in the latter in laminae, following two, less frequently three directions, parallel to each other and, evidently, definitely orientated with regard to the structure of ilmenorutile.

In the center ilmenorutile contains a few ilmenite inclusions, but towards the periphery the number of the latter is sharply increased and the outer portion of the specimen has usually the shape of an ilmenite border, while the upper portions of the crystals of the latter are frequently seen at the surface of hand specimens.

The author succeeded in discovering small ilmenorutile crystals in but one specimen. They have a prismatic shape; at the faces of the prism a striation is developed parallel to the axis of the prism.

The ilmenorutile from the 16[II] mine shows a conchoidal fracture; an indistinct cleavage is faintly marked along the prism. Under the microscope, however, two cleavages may be observed: one is clearly manifested, the other indistinct. The laminae of ilmenite are arranged parallel to the clearly visible cleavage.

The distinction between the streaks of ilmenorutile and ilmenite provides a perfect means to detect ilmenorutile in the as it were uniform hand specimen. Not only the ilmenorutiles from the Ilmen Mountains, but also those from Madagascar and Norway have a greyish-green streak.

The finding of ilmenorutile in pegmatites containing nephelite is a new fact, which up to now has not been mentioned in literature*.

Up to the present ilmenorutile was considered as a mineral of granite pegmatites and used to be encountered in paragenesis with topaz, phenacite, albite, quartz, beryl, cryolite and others.

Aside from the 16 [II] mine, ilmenorutile has been found in eight veins at the Lokhmataya, Firsovaya, and Savelyevaya mountains. Everywhere it occurs together with feldspar, phene, pyrochlor, hornblende, nephelite, biotite, occasionally calcium. A certain regularity may be observed in its distribution. It is encountered chiefly in pegmatites, poor in nephelite. On the other hand, in pegmatites rich in nephelite ilmenorutile has not been found in spite of the most thorough search, while ilmenite is abundant in these pegmatites. Considering the nephelite pegmatites as a later phase, and those containing no nephelite and syenite pegmatites as an earlier phase of a single pegmatite-forming process, the conclusion may be drawn that ilmenorutile began to crystallize out earlier than ilmenite.

It is of interest to note that ilmenorutile is found in the more eastern pegmatites at the Firsovaya, Lokhmataya and Savelyevaya mountains, just where pegmatites containing no nephelite are developed. Consequently, this part of the deposit may be regarded as being deeper than the western part (mine No 164).

Aside from primary veins ilmenorutile was detected in the black sands from Svistunov Log at the Vishnevy mountains.

Received
15. II. 1939.

* The intergrowth of ilmenite with rutile described by Panteleev for the Seliankin deposit seems to be actually an intergrowth of ilmenite with ilmenorutile.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

STRATIGRAPHY

ON THE AGE OF COAL-BEARING DEPOSITS OF THE WESTERN SLOPE OF THE MIDDLE URALS

By O. L. EINOR

(Communicated by A. A. Borissiak, Member of the Academy, 7.II.1939)

Coal-bearing deposits of the Carboniferous system are widespread on the territory of the Middle, and to a lesser extent, of the Northern and Southern Urals. Along the western slope the main area of their distribution lies in the districts of Kizel and Chusovaya of the Middle Urals. The age of the coal-bearing deposits within the boundaries of the Kizel district is defined in the recent work by I. I. Gorsky and I. L. Timofeeva as the uppermost Tournaisian up to the lower half of the Visean (*Seminula* zone of the Lower Carboniferous). According to the above authors, the upper series of the coal-bearing beds contains already Upper Visean fauna and corresponds to the base of the *Dibunophyllum* zone (D_1). The fauna of the upper part of the coal-bearing deposits is closely related to the fauna of the overlying limestones which belong to the open sea facies. According to the data of the majority of explorers of the Kizel district, the transition from the Upper Tournaisian marine beds to the coal-bearing deposits, and subsequently from the latter to marine limestones of the Visean, proceeds gradually, with no traces of break in sedimentation.

As to the Chusovaya district, there are indications to coal-bearing deposits lying on various horizons of the Tournaisian and even on the Upper Devonian.

Geological explorations, carried out by the author in the Kizel and Chusovaya districts during the period 1930—35, yielded new data allowing to determine with more accuracy the age of the coal-bearing deposits within the boundaries of certain areas of this rather vast region, the most important materials having been obtained for its least known southern part.

In the southern part of the Kizelovsk district on the territory between the rivers Usva and Vilva, the coal-bearing deposits are underlain by a series of clays, calcareous shales, hornfels and limestones rather variable along the strike (²). In some of the sections immediately beneath the coal-bearing beds are lying limestone beds with *Productus hyperboreus* Nal. etc., or siliceous shales, containing *Syringopora capillacea* Ludv. and *Syr. conferta* Keys. The transition from the described strata with Upper Tournaisian fauna to the coal-bearing beds in this district is quite gradual.

Much further south in the disctrict of the former Kynov region the coal-bearing beds are evidently reposing on the eroded surface of different horizons

of the Tournaisian. Within the boundaries of a relatively small area they are underlain by the following different horizons and facies:

a) syringopora horizon with *Syringopora capillacea* Ludv., *Syr. parallela* Fisch., *Syr. ramulosa* Gold., *Syr. gracilis* Keys, we believe to be synchronous with the *Productus hyperboreus* horizon of the Kizel district (uppermost Tournaisian);

b) *Spirifer tornacensis* horizon, referring to the upper part of the Lower Tournaisian;

c) horizon with *Martiniopsis waschkuricus* Frcks., *Spirifer medius* Lebed., *Cleiothyris royssii* l'Ev., *Chonetes laguessiana* Kon., *Productus* cf. *gorskyi* Nal., *Pr. laevicostus* Whit.

Still farther south, in the Staro-Utkinsky plant region, in the section along the Chusovaya river (left bank, above the village Rodina) the coal-bearing beds are underlain by limestones, carrying fauna as follows: *Lithostrotion* sp., according to I. I. Gorski's definition, *Syringopora* aff. *reticulata* Goldf., *Productus* aff. *concinnus* Sow., *Allorisma regularis* King. The occurrence of *Pr.* aff. *concinnus* Sow. and, particulary, of *Lithostrotion* sp. in the latter strata points definitely to the Visean age of the limestones underlying the coal-bearing beds. This being the first find of Visean index fossils in underlying deposits, there is ample reason to believe even the lower strata of the coal-bearing beds on the entire territory of the Chusovaya district to be Visean in age.

The upper border of the coal-bearing beds as distinguished from the lower one is in the region of the Chusovaya river, as well as in the Kizel district, strikingly constant and should be considered by us as lying on the same level. In the few sections of the Kizel district, where one may observe the direct contact between the coal-bearing beds and the overlying deposits, the latter are represented, as a rule, by the coral-brachiopod horizon. These strata abounding in fauna contain: *Productus mirus* Frcks., *Pr. probus* Rot., *Lithostrotion irregulare* Ed. et H. etc.

Less frequently the overlying beds are represented by dolomite and magnesian limestone, destitute of fauna; these layers evidently present a facial equivalent of the coral-brachiopod beds.

Finally, in single sections the coal-bearing beds are overlain by strata containing fauna monotonous and poor in species, occasionally comprising, for instance, *Productus* sp. aff. *striatus* Fisch., elsewhere *Chonetes papilionacea* Phill. (in prodigious numbers), or any other forms ([1], [2]). A detailed study of sections of the limestone beds overlying the coal-bearing deposits of the Lower Carboniferous, carried out by the author, leads to the conclusion that the lower strata of the beds, not always represented, as we have seen, by uniform rocks and fauna, are different facies of the same horizon.

This type of sections of the transitory layers persist over an extensive area southward along the middle course of the Chusovaya river. In the vicinity of the Kynov plant the sandstones of the coal bearing beds are overlain by limestones with *Pr. tenuistriatus* Vern., *Pr.* aff. *tenuistriatiformis* Einor, *Pr. mirus* Frcks., *Pr.* aff. *corrugatohemisphaericus* Vaugh., *Lithostrotion irregulare* Phill. etc., substituted in the direction of the strike with yellow coarse-grained dolomites. In the upper strata of the coal-bearing beds, among the interbedded almost black limestones, containing considerable admixtures of terrigenous material, we succeeded in discovering marine fauna wholly identical with the one widely spread in the overlying limestones, namely: *Productus mirus* Frcks., *Lithostrotion* sp., *Syringopora* sp.

The same faunal complex characterizes the lower strata of the limestone beds in the vicinity of Staro-Utkinsky district, i. e. in the southern part of the region covered by our investigations, near the southern border of the

range of coal-bearing deposits on the western slope of the Urals. In addition to the enumerated forms here are found: *Productus* aff. *hemisphaericus* Sow., *Pr. probus* Rotay, *Pr. continentalis* Tornq., *Syringopora parallela* Fisch., etc.

We may state that the succession of terrigenous sediments of coal-bearing beds upon marine limestone facies of the Tournaisian, in connexion with the uplift of the sea floor, proceeded irregularly and evidently not simultaneously at various sites of the region investigated. A considerable uplift of the bottom in some areas called forth conditions, under which accumulation of sediments was either suspended for a rather long period of time, or else superseded by erosion. Considering that on geographically rather limited areas coal-bearing deposits are underlain by different horizons of the Tournaisian, ranging from the top of the stage down to its lower half (Kyn), we are inclined to assume here rather the erosion on the face of the section of the Tournaisian, than the prevalence of conditions, under which no sedimentation was taking place, as has been suggested in literature. These events, which coincide with the epeirogenic uplifting of the sea bottom above the sea level, are referred by us to the very beginning of the Visean.

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Received
15. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GENETICS

EFFECT OF THE «INERT» REGIONS OF CHROMOSOMES ON THE EXPRESSION OF MOSAIC CHARACTERS

By N. I. NOUJDIN

(Communicated by N. I. Vavilov, Member of the Academy, 8.II.1939)

In our preceding paper ([3]) we have presented data concerning the influence of heterochromatic («inert») regions of chromosomes on the expression of mosaic characters in *Drosophila melanogaster*. These data have clearly shown that the so-called «inert» regions play an important rôle in the hereditary variation. Therefore we have come to the conclusion that the terms «genetically active» and «genetically inert», as applied to different chromosome regions ([8]), have lost their meaning.

The above conclusion was already prompted by the results obtained by a number of investigators ([2],[3],[4],[10],[11]), who have shown the transfer of an «inert» region of a chromosome into an «active» region to result in a modification of the properties of the genes located in the active region which, owing to rearrangement, happened to be in the proximity of the «inert» region. This conclusion is given especially sound foundation in result of our investigation([3]) in which we have shown that:

1. When forming new connexions with active sections of chromosomes, heterochromatic regions «throw» the genes located in their proximity into a state of increased lability or mutability (cases described by many authors). Not all parts of the «inert» region, however, possess this property. A pronounced qualitative differentiation exists within it.

2. The heterochromatic regions of chromosomes («inert» regions of the *X*-chromosome, the *Y*-chromosome, «inert» region of the IV-chromosome) all suppress mosaicism of the «eversporting» type, which points to their genetic homology.

3. There exist a rather strict quantitative relation between the mosaic-suppressive action of the elements just mentioned.

4. The study of the influence exerted by additional heterochromatic regions on the development of mosaic characters has shown the very idea of mosaicism to be conventional. The expression of a mosaic character may vary within so wide a range that it may become indistinguishable from a character conditioned by a mutated gene (the «*A*» gene, f^{B15}, $sc^8w^af^{56}B$).

5. Heterochromatic regions of chromosomes do not only affect the frequency of occurrence of mosaic individuals and the degree of mosaicism of the individual, but the kind of mosaic characters (their quality) as well. Under the influence of different quantities of heterochromatin, mosaic characters

develop in the form of one or another member of the series of mutable allelomorphs known for the given gene.

6. The type of mosaicism (frequency of occurrence of mosaic individuals, degree of mosaicism in the individual and the form of expression of the mosaic character) depends upon the nature of the intrachromosomal rearrangement which gave rise to a given mosaic strain. Thus, in the original mosaic strain, *scute*-8, 16% of females are mosaic, and the tissues showing the mosaic character form isolated spots of different size on the body surface (2).

The strains obtained by M. L. Belgovsky and possessing mutations of the gene *yellow*, induced by X-raying of the *scute*-8 chromosome, proved to be mosaics, although of a type quite different from that of the original *scute*-8 stock. The following types of mosaicism for the *yellow* character are presently known to be conditioned by the *scute*-8 chromosome: a) The original *scute*-8 stock. Flies with normal body and wing colour. 16% of females having yellow spots of different size on the body surface.

b) Body surface and wings of the females, as well as the majority of macro- and microbristles, yellow, all the females having some normally coloured bristles (the y^{B270} stock).

c) Body, wing and bristle coloration in 99% of the flies yellow, the remaining flies having 1—2 normally coloured macrobristles (the y^{B2} stock). Hence it should be concluded that X-raying of the *scute*-8 chromosome may give rise to strains showing different types of mosaicism, the latter remaining constant in successive generations.

It is of interest to note that the influence of additional portions of heterochromatin on the expression of mosaic characters in those stocks is far from being similar. In the *scute*-8 stock in the presence of one of the arms of the *Y*-chromosome the ratio of mosaic females falls to 0.5, the area occupied by a mosaic spot sharply decreases, only 1—2 bristles remaining yellow. In the y^{B270} stock males distinctly differ from females, even without any additional heterochromatin being introduced into them. The chitinous parts of their body and their wings have a normal coloration. This peculiar sexual dimorphism depends upon the influence exerted by the *Y*-chromosome of the male. The introduction of one of the *Y*-chromosome arms to the females makes them indistinguishable from the males. It follows that in this stock additional portions of heterochromatin suppress the development of the yellow coloration in the wings and in the body and increases the number of normally coloured bristles, while all the individuals remain mosaic in respect to the bristle colour. In the y^{B2} stock additional heterochromatin does not suppress the development of the yellow coloration either in body or wings, but increases the ratio of individuals having normal bristles up to 11%. And finally in the y^{B396} stock the additional portions of heterochromatin do not exert any influence. It should also be noted that by X-raying males of the y^{B270} stock we have succeeded in producing further shifts toward a stronger expression of the *yellow* character.

All thes data allow us to conclude that the change of a normal gene into its recessive allelomorph may occur not only in the form of a sharp mutational change leading directly to a 100% manifestation of the character on hand, but also in the form of a series of slighter changes, these leading, however, eventually to the same results. The *yellow* character under consideration would seem to result from an integration of a series of hereditary changes. Some of them affect only single bristles, further affect the colour of the chitinous parts of the body, that of the wings and of a part of the bristles, then changes occur which give strains of yellow flies with a few individuals having 1—2 normal bristles, and finally strains are produced, in which the manifestation of the character in question is practically complete. It seems

quite possible that the members of the series of multiple allelomorphs may also be regarded as successive steps in the process of hereditary variations. In this connexion we consider it necessary to point to the work of Jollos (1), who obtained analogous data which made him assume the existence of «directed» variation.

It seems to us that data we have presented are noteworthy and permit of drawing some conclusions relating not only to the question of the nature of mosaicism. It is presently well known that mosaicism is very frequent both among animals and plants. Many phenomena (mutable genes, position effect) are known to be in their nature very closely related to mosaicism. On the other hand, it is known that heterochromatic regions are not specific for *Drosophila* chromosomes. They are known also in plants and probably constitute a component of any chromosome. Furthermore, A. A. Prokofieva-Belgovskaja (13) has discovered 10 separate heterochromatic regions in the *X*-chromosome of *Drosophila melanogaster*. All this makes it necessary to revise our conception of the structure of chromosomes, of the rôle played by «inert» and «active» regions and of the nature of the gene itself.

In our preceding paper we have pointed out that the experimental data now at hand show that there is no difference between mutations, the phenomenon of position effect, and the mosaicism of the type concerned. The evidences presented in paragraph 6 of the present article are still more convincing in this respect.

The above data are in a good agreement with the views advanced by Morgan, Bridges and Schultz (7). From our data these authors come to the assumption that heterochromatic regions are concerned in the formation of some substance, which these authors inclined to identify with thymonucleic acid. In a more recent paper by Carperson and Schultz (5) this conception is worked out in more detail. If the rôle played by the heterochromatic regions in mosaicism is to be attributed to their capacity of producing some definite substance (possibly, the thymonucleic acid), our data on the paternal effect (3) deserve special attention. If such a substance is actually produced, it must be formed at all stages of development, i. e. in ovogenesis, in spermatogenesis and in somatogenesis, and our experiments have shown that this is actually the case.

Muller, Prokofieva and Raffel (9) have studied 27 mutations in the *achaete* and *scute* loci and shown 18 of them to be linked with minute intrachromosomal rearrangements. Considering these data and keeping in mind the difficulty of detecting the rearrangements of that kind cytologically, we can safely assume that the majority of mutations are due to intrachromosomal changes of this type. These rearrangements lead to the formation of new intrachromosomal connexions presumably of chemical nature, this being the cause of the modifications in the expression of hereditary characters observed by us. This assumption is in complete agreement with our conception as to the lack of essential difference between mutations, on the one hand, and the phenomena of position effect and of mosaicism, on the other.

Cytological studies of chromosomes along with Casperson's data show that the chemical nature of «inert» and «active» regions is different. We may assume that the discs observed in salivary gland chromosomes and the «inert» regions of chromosomes have a common nature, being virtually identical. Consequently, the «active» regions must consist of heterochromatic and euchromatic sections. From that point of view it becomes comprehensible why a transfer of a heterochromatic chromosome section into an «active» region induces position effect or mosaicism, while a rearrangement involving «active» regions alone results in position effect or a mutation. On the same basis we may explain the strict quantitative regularity in the influence of heterochro-

matin on mosaic characters, the ability of heterochromatin qualitatively to modify the expression of mosaic characters, which results in the appearance of a series of multiple allelomorphs, the differences in the degree of mosaicism, etc. All this is due to the fact that there is a common cause responsible for all the phenomena described, viz. the interaction of heterochromatic and euchromatic sections of the chromosomes, which result in the quantitative and qualitative modifications observed.

The conception here discussed is in a complete agreement with the views of Wrinch, Goldschmidt ([6], [12]) and Shmuck who regard the chromosome as a giant chain molecul. We must pay particular attention to the above-mentioned article by Shmuck. The ideas developed by this author allow us to understand how changes which led to the formation of new intrachromosomal connexion result in the origination of new, qualitatively different sections of chromosomes, capable of addition of different amino-acids synthesized in the cell.

In connexion herewith we must emphasize the great rôle played by the cell as a whole in the determination of hereditary traits, as well as the rôle of external conditions (in the broadest sense), because the synthesis of the compound set of amino-acids present in the cell is determined by the nature of the historically formed system of the cell itself and by the external conditions under which the development takes place.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GENETICS

ON THE NATURE OF THE SO-CALLED INERT REGIONS OF THE CHROMOSOMES IN RELATION TO THE PROBLEM OF THE CHROMOSOME STRUCTURE

By JUL. KERKIS

(Communicated by N. I. Vavilov, Member of the Academy, 9.II.1939)

The discovery by Muller and Painter in 1932 ([14]) of the so-called genetically inert region in the proximal portion of the sex-chromosome of *Drosophila* started a whole series of investigations devoted both to the study of cytological structure of this region and to elucidation of its rôle in heredity. Following the cited investigation by Muller and Painter, works appeared which proved that the «inert» regions constitute a significant part of the *Y*-chromosome in *Drosophila*, that they are present in the autosomes of *Drosophila* near the spindle fiber attachment, as well as in the chromosomes of certain plants ([13]). Finally, in her thorough investigations Prokofieva-Belgovskaja ([6–9]) found such regions not only in portions adjacent to the spindle fiber attachment, but their being also scattered at several points along the length of the chromosomes between their «active» regions. It obviously follows from the literature devoted to the «inert» regions that this term by itself is substantially erroneous. The «inert» regions proved to be of great importance in a whole series of phenomena of heredity, and it seems hardly possible at present to contrast them in this respect to the «active» ones. Upon closer examination of the well-known facts it becomes clear that the question on the «inert» regions is not a question of mere terminology and that the time has arrived to critically revise the current idea as to the nature of the «inert» regions; further approach to their study should depend on the results of such an analysis.

As a result of a discussion of the problems connected with the «inert» regions, which are being so very successfully studied cytologically by A. A. Prokofieva-Belgovskaja at the Institute of Genetics of the Academy of Sciences, I have arrived at a conclusion that these «regions» should be interpreted in a different manner than it is being done by Prokofieva ([6,7,8,9]).

It was shown as early as in 1881 by Balbiani, who was the first to describe the specifically characteristic nuclear structures in the salivary glands of *Diptera*, that the chromosomes of these nuclei consist of alternating portions of chromatic and non-chromatic substance which was termed by him interstitial substance («Zwischensubstanz»). Heitz later suggested to call the substance of the chromatine disks euchromatin and the «interstitial» substance—heterochromatin.

It seems to us that the «inert» regions of the chromosomes do not exist in the form of «specific portion of the chromonema with chromioles» as describet by Prokofieva, since on principle there is no difference in the structure of the «active» and «inert» regions. Regions described as «inert» correspond

to those, where chromioles are far separated from each other or are so small that such portions seem to contain no clear-cut disks. The substance of the «inert» regions is apparently identical with the interstitial substance of Balbiani or with the Heitz' heterochromatin. This substance, according to all the evidence available, is different in different parts of the chromosome and may sometimes constitute a considerable part of the chromosome's total length. The chromioles are arranged in it in linear order. As the chromosome in the salivary glands appears to be a bunch of genonemata which conjugate parallel, it is conceivable that the interstitial or heterochromatic substance is identical with the genoneme portions located between the chromioles linearly arranged in it.

On the basis of his extensive experiments Noujdin ([4],[5]) arrived at the conclusion that the substance which is tentatively called by us interstitial substance is not merely a matrix, in or upon which lie the chromioles (genes?) which are functionally independent of it, but, on the contrary, that the interrelation between the interstitial substance and the substance of the chromioles is apparently such a close one that attempts to identify genes merely as one of these two substances are hardly correct. This idea developed by Noujdin on the basis of his data also follows from Goldschmidt's idea ([12]), who interprets the chromosome as a giant protein molecule, as well as from the literary data on differential influence of the interstitial substance upon the property of the genes connected with it ([10],[15])*.

All the properties of the «inert» regions described up to the present day may be accounted for on the assumption of the hypothesis developed by us above. One of these specific properties of the «inert» regions is, following to Prokofieva-Belgovskaja ([6],[8]), their ability to conjugate with each other, as well as with the «regions» located in the vicinity of the points of the spindle fiber attachment. As conjugation of chromioles takes place, the phenomenon in question differs in no way from ordinary conjugation of any «active» portion with another homologous one. Nobody has proved yet that all the «inert» regions described by Prokofieva-Belgovskaja and other investigators either at the ends or in the middle of the chromosome do always conjugate with each other. This phenomenon may be, therefore, successfully explained in the same way as conjugation of the two active regions in the same chromosome, i. e. by the presence of duplications ([3],[11]). One could speak of a specific affinity between different «inert» regions in case it were shown that conjugation takes place not between the chromioles of these regions, these chromioles differing in nothing, as Prokofieva-Belgovskaja argues herself, from those of the «active» regions, but the «inert substance» itself which forms these portions. However, it has not been done yet. According to observations by Prokofieva-Belgovskaja, the chromosome portions described by her as being «inert» represent a kind of «weak points», since the chromosome most often breaks or bends there. This phenomenon may be fairly well explained (as admitted also by Prokofieva-Belgovskaja) by the mechanical properties of the interstitial substance which even by visual investigation under the microscope appears to be considerably less dense than the substance of the chromioles.

The conclusion arrived at by Belgovsky ([1]) is in accord with our hypothesis too; this author, who studied the properties of the genes located in the «inert» regions or in their immediate proximity, concluded that genes of the «active» regions on principle are in no way different from the genes of the «inert» regions of the chromosomes.

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* After this article was ready, there appeared a detailed account of the work by J. Schultz and T. Casperson ([17]), which prove the influence of interstitial (heterochromatic) substance upon the properties of neighbouring genes.

Koltzoff, Goldschmidt and Wrinch ([2],[12],[16]) have independently worked out an idea of the chromosome, as a giant protein chain molecule, whose axis (the genonema) consists of polypeptid links, to which at different points are attached the lateral chains (genes) composed of protein particles. If this conception be true, the viewpoint advanced by us, i. e. that the chromosome is composed of the chromiole substance and of the interstitial substance, hitherto called «inert», may find support in the chemical model of the chromosome, described by the above authors. A mere assumption would suffice that the distances between the side chains, which are most active chemically and correspond, according to Koltzoff, Wrinch and Goldschmidt, to genes, differ along the length of the chain and form these spaces between the chromioles which we arbitrarily call the interstitial substance. The length and qualitative properties of these interstitial links determine a number of properties of the protein radicals attached to their ends. The activity of side chains and of the interstitial links is specific in such a chain molecule, so that in this respect the chemical model of the chromosome conforms chemically to the hypothesis advanced by us above. The chain molecule should, therefore, be differentiated along its length into qualitatively differing portions of various length. If some of the side chains along such an axis prove as similar as to possess specific ability to attract each other, than the whole chain bends correspondingly and these similar radicals come into contact, conjugation of the homologous sections thus taking place. The point of view abvanced by us on the nature of the so-called «inert» regions thus proves in a sufficient agreement with the idea of the chromosome as of a giant chain protein molecule, to justify the above juxstaposition. However, in case further scientific developments prove the above described chemical model of the chromosome not to be true, this should in no way disprove our idea of the «inert» regions of the chromosomes, as it has been developed quite independently of this model*.

In case the correctness of this model and of the above reasoning will be supported, a whole number of phenomena of chromosome behaviour (crossing over, formation of various structural rearrangements and configurations), whose mechanism is not yet quite understood, will receive comparatively simple explanation on the ground of the physico-chemical properties of the chain molecules.

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* After the present paper was ready, A. A. Shmuck made me acquainted with the manuscript of his paper, in which he supports and develops the idea of the chromosome as of a chain molecule and defines in concrete chemical terms the viewpoint on the nature of the «inert» regions advanced by me.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GENETICS

HIGHLY MUTABLE STOCK FROM A WILD POPULATION OF *DROSOPHILA MELANOGASTER*

By G. G. TINIAKOV

(Communicated by A. A. Sapehin, Member of the Academy of the Ukrainian SSR, 14.II.1939)

The first publications on mutable stocks from normal and wild populations of *Drosophila* apppeared in 1937 ([1],[2],[3]). As to the unpublished data obtained in this country, the following authors should be cited: R. L. Berg, S. M. Gerschenson (the latter reported cases of frequent occurrence in nature of the character *yellow* without attributing these cases to the direct appearence of mutations), J. M. Oleneva, N. D. Duseeva, N. N. Sokolov and N. T. Andrushina-Rybakova.

Although the problem of mutable stock in wild populations is of recent origin, it is of great importance, since the existence of mutable stocks in nature throws new light on the theoretical problems of populations and on the complicated problem of the nature of the mutation process in general. Moreover, this fact brings material changes into the current views as to the expansion capacity, character and frequency of mutations among natural populations, thus adding new data concerning the problem of evolution.

In the present publication a brief account is given of the formation of a highly mutable stock of *D. melanogaster* recently discovered by the writer, as well as a description of a set of mutations detected in this stock.

In fall 1937 we were collecting wild *Drosophila* populations in the Moscow district. On September 25 5 females and 4 males of *D. melanogaster* were found caught in our vials in a vegetable garden in Khotkovo (60 km from Moscow).

Four females and four males had a normal phenotype, one female had narrower eyes, reminding the character *kidney* or a faint expression of the heterozygous *Bar*. This female gave in F_1 9 *Bar* females, 13 normal females and 11 males possessing the character *yellow*. This sex ratio in F_1 and in the subsequent generations and the occurrence of this character in the females alone pointed to that this character is actually the character *Bar*, which is lethal in homo- and hemizygous condition. It was assumed that the wild population has been accidentally contaminated by this character through the Moscow stocks. This stock was cultivated at room temperature till August 1938. The remaining 4 normal males and 4 females were also crossed; they gave a well propagating stock of normal flies of the Khotkovo population. This stock was also kept at room temperature.

Till August 1938 both the *Bar* stock, which was continuously mated with normal males, and the normal Khotkovo stock gave several times

single males *yellow*, but no special importance was attached to this fact. The Khotkovo *Bar* stock perished due to hot summer and in but one of the vials normal flies survived, viz. two females and three males phenotypically normal. These 5 flies gave rise to a new normal stock of the Khotkovo population. At the end of October 1938 in one of the vials one *Bar* female was found again.

The reoccurrence of *Bar* in a single female per 217 females was most surprising, the more so as besides *Bar* pronounced *plexus* and blistered wings were found in 37 females and 42 males out of 438 flies from mass cultures examined. Individual crosses of the new *Bar* stock, the *plexus* stock and the normal original stock were made. The material was obtained from three generations during a month and a half. During this period a total of 8634 flies were carefully examined and among them 54 distinct different variations were found, the majority of which were apparently mutations.

The character and frequency of occurrence of separate characters are given in the table.

Character and Frequency of Occurrence of Mutations in the Khotkovo Mutable Stock per 8634 Flies Examined

№	Character of the mutation	Frequency of occurrence of the mutation			Dominant or recessive character
		♀♀	♂♂	Total	
1	*Bar*	2	—	2	Dominant
2	*Yellow* body and bristles	—	6	6	Recessive
3	Greatly reduced eyes, fused facets, changed eye colour (*lozenge*)	—	3	3	»
4	Greatly reduced wings (*miniature*)	—	2	2	»
5	*Forked* bristles	—	1	1	»
6	*Plexus*	1	1	2	Semi-dominant
7	Divergent wings	1	1	2	Recessive
8	Blistered wings	1	1	2	»
9	Velvet folded wings, different proportions of body	1	1	2	»
10	Brushes on the wings (at the base)	3	2	5	?
11	Horn-like protrusions on the sides of the body at the base of the wings	—	6	6	?
12	Greatly reduced flattened eyes (*Lobe*)	2	—	2	?
13	Two pairs of wings, the second pair of well developed wings has formed from the halters (*tetraptera*)	—	1	1	Recessive
14	Lobed wings. The lobes form at the base of the wings as large rounded formations surrounded by a marginal vein	—	2	2	?
15	Dark body colour (*ebony* or *black*)	1	1	2	?
16	Rough eyes	1	2	3	?
17	Black chitin formations at the ends of the wings	1	1	2	?
18	Black shortened antennae	—	4	4	?
19	Absence of anterior ocellar bristles (*scute*)	1	1	2	Recessive
20	Cutaneous protrusions above the aristae	1	—	1	?
21	Aristae transformed into paws (*aristopedia*)	—	1	1	Recessive
22—43	Different independent wing mutations	39	31	70	?
44—47	Different eye changes	5	5	10	?
48—51	Different body mutations	15	11	26	?
52—54	Other bristle changes	9	3	12	?
	Total	84	87	171	

The table illustrates the great mutability of the stock under consideration. The variety of mutations, their frequent reoccurrence, the occurrence of very rare and in some cases perhaps even unknown mutations is striking.

In usual wild and laboratory stocks of *D. melanogaster* ([4]) and *D. funebris* ([5]) there occur as a rule 5 visible mutations per 100 000 flies. The spontaneous frequency of occurrence of visible mutations in our mutable stock surpasses therefore the usual frequency 125 times. If the recurrent cases are taken into account, the frequency of occurrence of visibles will apparently 400 times surpass the usual frequency. Since, however, some errors were possibly made in recording the recurrent cases, because part of the material was obtained on mass cultures, and since the stocks *Bar* and *plexus* were obtained from a normal mass culture, part of the characters might have already been multiplied, being thus mistaken for recurrent ones. The above comparison is therefore to be considered as an approximate one. As to mutations *lozenge*, *miniature*, *forked* and others, these were always found in the ratio of one to 200—300 individuals obtained from one pair. Besides, the individuals found in the stock *plexus* always carried the character *plexus* as well. For instance, 4 males *yellow* and 2 males *lozenge* appearing in the stock *plexus* phenotypically were also *plexus*. Thus, the reoccurrence indicated in the table is fully significant with regard to these and a number of other mutations.

It is noteworthy that in this mutable stock such rare mutations, as for instance *tetraptera*, were found (absent since 1929 in the stocks of the Institute of Experimental Biology; it has appeared in the stock *plexus*, carrying the character *plexus* on its four wings too). The same should be said of *Bar*, *aristopedia*, *lozenge* and others.

Such mutations as lobular wings, brushes on the wings, horn-like protrusions on the sides of the body, chitin formations on the ends of the wings etc., are possibly quite new. Judging by the peculiarities mentioned above and the extremely high mutability, the mutability of our stock is apparently the highest of all the spontaneous mutable stocks of *Drosophila* so far known [according to Plough and Holthausen ([1]), their mutable stock by 3—6 times surpasses the common mutation frequency]. But at the same time it is striking that all these mutable stocks are apparently of rather wide occurrence and of common nature, because in all the cases known they begin first of all by the appearance of *plexus*, blistered, divergent wings, *abnormal abdomen*, and others.

A preliminary cytological analysis of the salivary glands of this stock showed that a disturbance in the chromosome conjugation is of frequent occurrence. Along with this we should mention non-homologous conjugation of the type discovered by McClintock ([6]) on maize, conjugation of duplication type in one haploid, as established by Bridges ([7]), and chromosome rearrangement (two cases in our preliminary material). All this points to the fact that the study of the mutation process will very soon gain due attention in connexion with conjugation and other kinds of chromosome behaviour, as dependent on the internal and external factors.

A detailed cytogenetic analysis will be given elsewhere. It should be once more emphasized that the majority of the changes obtained are mutations, because of the 54 cases mentioned 16 changes have already been isolated as well propagating independent stocks. Here belong: 1) Bar^{207}; 2) *plexus*, 3) *yellow*, 4) divergent wings, 5) blistered wings, 6) *plexus* near the small transversal, 7) velvet wings, 8) *delta*, 9) *scute*, 10) *lozenge*, 11) *tetraptera*, 12) *minute*, 13) *forked*, 14) Bar^{28}, 15) *cut*, 16) *abnormal abdomen*.

About 10 000 flies from wild populations of Topornino (near Ufa) and Polivanovo (170 km to the south of Moscow) served as control. These stocks

were placed since the fall 1937 under the same conditions as the experimental flies. Except several faint deviations these 10 000 flies proved normal.

An extensive study of this highly mutable stock is now in progress.

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Received
15. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GENETICS

IMPROVEMENT OF VARIETIES OF INDIAN CORN THROUGH INBREEDING COMBINED WITH CROSS-POLLINATION

By M. L. KARP

(Communicated by A. A. Sapehin, Member of the Academy of the Ukrainian SSR, 14.II.1939)

Our experimental studies ([1],[2]), as well as practical breeding of corn in the USA ([3],[4],[5]), have shown that crossing of an open-pollinated variety with an inbred line, the so-called top-cross, may help to improve a variety even in that case, when the inbred line applied is of lower quality than the open-pollinated variety. Genetically considered the sense of this cross consists in selecting the very best gamete with the aid of isolation through inbreeding in order to use it as partner for each of the gametes of the population.

We have similarly shown that selection of material valuable for inbreeding is considerably easier when the F_1 from the top-cross is used, not the open-pollinated variety. All the plants belonging to the F_1 from the top-cross, as well as the average families from the first generation obtained from inbreeding these plants, are identical with regard to the one half of their chromosome set derived from the inbred line; they differ only in those chromosomes which came from the population. We are able, therefore, to analyze a great number of gametes with the aid of the top-cross and to start inbreeding by selection of material which is more valuable for inbreeding.

I wish to show in the following an additional possibility which is opened by this cross and which in our opinion is of great interest to breeders. We think of the possibility of a radical improvement of a variety in a comparatively short period of time, viz. in 2 or 3 years, by selecting the best families of the 1st generation of inbreeding applied to the top-cross material and by subjecting these families further to open pollination.

Hayes and Garber ([6]) were the first to advance the idea of using mixture of hybrids between different inbred lines (multiple cross) for production of a new variety. Hayes ([7]) has shown further that such a synthetic variety may considerably surpass in yield the original open-pollinated varieties. In the USA the synthetic varieties of Indian corn are introduced at present in the practice; they are distinguished by high yield and by other valuable qualities ([4],[5]).

The open pollination among the first generation of inbreeding is also directed toward synthesis of the most valuable genotypes. At the same time our method materially differs from the multiple cross method which is the foundation of synthesizing new variety.

We need not practice inbreeding during a long period of time in order to select valuable genotypes. When material obtained from top-cross is used for inbreeding, such a selection as pointed out before may be no less successfully done in the 1st generation of inbreeding. Owing to the fact that isolation may be once done without great expenditures even on a large material, up to several tens of thousands of plants, combination of the most valuable gametes derived from an open-pollinated variety may be conducted on a very broad genetical basis. Such a method should not necessarily result in production of a new variety, but may only lead to improvement of an already existing one. Properties characteristic of a given variety may be preserved as well as its high heterogeneity; at the same time individual characteristics of the variety and its total productivity may be considerably improved.

The system of renovation of the varieties of Indian corn advanced by us is thus designed for 3 generations.

First generation—crossing an open-pollinated variety with the best inbred line. This crossing may by itself lead to improvement of the variety.

Second generation—isolation of a great number of plants obtained from the top-cross mentioned above. Selecting material for inbreeding from the F_1 produced by the top-cross, one attains possibility of producing highly valuable inbred lines.

Third generation—open cross-pollination among the best families of the first inbred generation from plants obtained from the top-cross. This crossing is based upon selection of best gametes and should result in a radical improvement of the variety.

We are convinced that the system advanced by us will prove justified not only in the case of Indian corn, but in other cross-pollinated plants also. In corn we are ready to start work of practical importance. In order to obtain sufficiently productive inbred lines in other crops, preparatory work is necessary.

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Received
15. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GENETICS

A CONTRIBUTION TO THE PROBLEM OF INTERSEXUALITY IN *LYMANTRIA DISPAR* L.

By P. A. KOSMINSKY

(Communicated by I. I. Schmalhausen, Member of the Academy, 19.II. 1939)

In a series of papers (1) I had presented the results of my studies on two lines of *Lymantria dispar* which produced in subsequent generations in every brood either normal females alone or both normal females and male mosaics uniting male and female characters or, finally, normal females and males, both normal and mosaics. On the basis of my studies I arrived at a conclusion that ability of producing male mosaics was transmitted through the mother and that male mosaics were intersexes; however, a number of peculiarities observed in them could not be explained by the so-called «time law of intersexuality» of Goldschmidt; according to this law, modification of the organs should proceed in a definite order, viz., the later differentiation of an organ takes place, the easier is this organ modified.

Goldschmidt, while discussing the data obtained by me, was several times changing his opinion as to the relation between the phenomena observed by me and intersexuality; it was dependent upon whether or no he succeeded in finding a hypothesis suitable for explanation of the occurrence of mosaics on the basis of his general theory. In his last work (3) he arrives at a final conclusion that the mosaic-producing lines found by me have nothing to do with true intersexuality and that facts presented in my paper, while they are inexplicable from the «time law» view point, are nevertheless not in conflict with this law.

Goldschmidt bases his conclusions upon the following statements:

1. The proofs for the transmission through the mother of the ability of producing mosaics «are very meager, if not absent»; the origin of such lines is due to mutation of autosomal genes, and various grades of expression and manifestation depend upon a series of modifiers. 2. In identical crosses in my lines various results with regard to expression and manifestation are obtained, which is never the case in true intersexes. 3. Definite difference between my mosaics and true intersexes is observed.

As in my previous works I have not discussed the question on the possibility of producing mosaics as a result of autosomal mutations, I take it to be necessary to dwell upon it now in order to present reasons demonstrating the inadmissibility of Goldschmidt's hypothesis. My statement that the ability of producing mosaics is transmitted only through the mother Goldschmidt considers to be questionable, since the fact that normal progenies occurred from 16 crosses of male mosaics with females from normal lines,

while mosaics were constantly obtained in the progenies from crosses of females from mosaic lines with males from normal lines, may be explained by the presence of a certain number of autosomal genes, one of which in homozygous conditions is necessary for manifestation of mosaicism.

Goldschmidt thinks that in my exceptional lines may be assumed the presence of a lethal gene which is peculiar only to them and is limited to the male sex; a certain number of modifiers should also be assumed. He tries to make fit this scheme for some of my crosses.

The results of a series of our crosses may fit the scheme which assumes the presence of autosomal mutant genes in my exceptional lines; to make them do so it is necessary to assume the presence of an indefinite number of modifiers and to postulate that the gene peculiar to our exceptional lines and effecting the occurrence of mosaics is not a lethal, but that the death of males takes place in case of a definite combination of modifiers; finally, it should be assumed that part of the modifiers are contained in one race, the other part—in another.

However, we possess data which under no circumstance can be explained in this way.

No matter which hypothesis is admitted, we are compelled to assume that our exceptional lines differ from the rest of the normal lines in possessing a special gene, in the absence of which mosaics cannot be produced, since mosaics were obtained only from crosses with these lines.

Let us consider from this viewpoint one series of crosses in one of my lines (from Poltava).

Females from the hatch containing a great excess of females (the properties of the males remained unknown, since they were fixed for histological purposes at the caterpillar stage) were crossed with normal males from a population and gave mosaics in the progeny (1st generation). Females from this progeny were crossed with males from a normal hatch consisting of a great number of individuals (about 300), obtained from a cross between individuals from normal hatches; two normal hatches and 13 hatches containing mosaics or consisting only of females were obtained (5 additional hatches obtained contained extremely low numbers of individuals) (2nd generation).

Females from normal hatches of this generation crossed with normal males from checked* normal lines gave mosaics in the progeny. All the females from the two hatches which produced only females were crossed with males from normal hatches. In one case 46 hatches were obtained. The majority of them gave only females or both females and mosaics; only in four cases ratio of the male mosaics to normal ones could be (not without difficulty) considered to answer the 1 : 1 ratio (1 normal : 5 mosaics; 1 normal: 4 mosaics; 8 normal : 8 mosaics; 12 normal : 12 mosaics); in another case 20 hatches were obtained which also contained only females or an excess in females plus mosaics, and only in two hatches the ratio of male mosaics to normal ones was close to 1 : 1 (22 normal : 27 mosaics; 12 normal : 5 mosaics) (3rd generation).

10 females from the one of the hatches of these series crossed with normal males from the lines checked gave in 9 cases mosaics, and in one case 5 ♀ and one normal ♂; this indicated an abnormal ratio, since when dying out was strong an excess of females was always produced (4th generation).

If we assume that the original females were homozygous for the basic gene which is necessary for the manifestation of mosaicism, we should obtain in crosses with normal males females of the genotype *Ll* (we denote this

* They were checked during three generations of inbreeding and by crossing with normal lines of other races.

basic gene after Goldschmidt by the capital L). In the progeny of such females we should obtain, upon crossing with normal males, in each hatch half of the individuals with the genotype Ll and the other half with ll, i. e. each hatch should consist of equal numbers of normal males and of mosaics. As a matter of fact in the second generation we have obtained only females in 4 hatches ouf of 15. From crossing all the females of a hatch of the second generation with normal males we should have obtained in the third generation normal progeny in one half of the hatches, while the other should have consisted of equal parts of normal and mosaic males; however, this does not answer the actual situation at all.

One may assume that normal males taken for crossing with original females (first generation) from a population possessed the gene L, but mosaicism was not manifested in them, owing to absence of the necessary modifiers; however, in this case we should have expected in the third generation in each hatch half of the males to be normal, while in the fourth generation half of the hatches should have been normal which was actually not obtained.

It could, finally, be supposed that the normal line used in crosses for obtaining second generation was not entirely pure, so that males similarly contained the gene L which remained non-manifested, owing to absence of modifiers or owing to heterozygoty for that very modifier which should have been present in homozygous state for manifestation of mosaicism*. However, from crosses with such males we should have obtained either entirely normal progeny, or equal numbers of normal and mosaic males in each hatch; as a matter of fact, however, we obtained in a number of hatches only females, while others consisted only of mosaics.

Thus, even when we make a series of rather violent assumptions, we are still unable to explain phenomena observed by us through presence of autosomal mutations in my exceptional lines.

All the above data prove with sufficient conclusiveness that the ability of producing mosaics is transmitted exclusively through the mother.

The second distinction between my mosaics and true intersexes pointed out by Goldschmidt, viz., great diversity in identical crosses within my lines, may be easily explained by the presence of modifiers; moreover, as I have already pointed out in 1935, Goldschmidt himself has obtained a diversity similar to that in my material in each case, when repeated identical crosses were made (e. g., the cross between the race Toyama with the race from Berlin).

The third Goldschmidt's statement is disproved as pointed out earlier (1936) by complete absence of any distinction between my mosaics and intersexes obtained by me as a result of crossing with each other strong and weak race. The only difference noticed, as emphasized by Goldschmidt, consists in the absence in intersexes of modified hind outgrouths, while the anterior ones are normal; however, it is immaterial, for such a modification is rarely met with in my mosaics also **.

The statements of the similarity of the phenomena observed by me to the phenomena of intersexuality, made by me earlier, are thus not disproved by Goldschmidt's arguments. Goldschmidt himself admits that modifi-

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* Presence of such a modifier is necessary for the explanation of the fact that crossing of mosaic males with normal females produces normal progeny ([3]).

** Goldschmidt, referring to my paper (1935), points out, seeking for evidence in favour of the fundamental distinction between intersexes and my mosaics, that I have not received modification of antennae in male intersexes from crosses of the Japanese race Akita with the strong race. In further crosses I succeeded in obtaining changes of antennae of the very type as that in my mosaics; it occurred in individuals possessing comparatively weak modifications of other organs.

cations observed in my mosaics are inexplicable on the basis of the «time law».

A few words should be added regarding certain phenomena which, to my opinion, are inexplicable on the basis of the «time law». The following phenomena I include here: 1) absence of alterations in the ring of the 9th abdominal segment and in the saccus connected with it in «strong» male intersexes; 2) presence of female sex-cells in testes of female intersexes which possess normal male copulatory apparatus and lack bursa copulatrix; 3) the fact that in female intersexes of the type «Gifu» the rear row of outgrowths of the antennae is modified first, while in male intersexes modification begins in the anterior row.

Goldschmidt gives no comments on the last point. He explains the great stability of the 9th segment and of the saccus by admitting that organs, which have no homologues in the other sex, appear to be stable even when they are differentiated late. However, apophyses in females, which have no homology in males and develop like saccus, i. e. invagination of intersegmental fold, become modified even in lowest grades of intersexuality.

Presence of female sex cells in testes of «strong» female intersexes Goldschmidt explains by admitting that development up to a certain moment proceeded in the direction of femaleness; after this moment a part of female cells die, the rest surviving. However, the copulatory apparatus begins to be differentiated as early as prior to hatching of caterpillars; therefore, when the female copulatory apparatus is completely transformed into the male one, the «turning point» must lie in the embryonic stage, when there is no distinction between female and male sex cells.

All the other arguments of Goldschmidt were answered in my previous papers.

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Received
15. II. 1939.

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1939. Volume XXII, № 9

GENETICS

CHROMOSOMES OF THE TORTOISES *EMYS ORBICULARIS* LINN. AND *TESTUDO IBERA* PALL.

By G. M. PKHAKADZE

(Communicated by N. I. Vavilov, Member of the Academy, 24.I.1939)

Up to the present day the chromosomal complexes of turtles (*Chelonia*) which belong to one of the orders of *Reptilia* were very little investigated cytologically. The available cytological observations on chromosomes of *Reptilia* are of occasional character and refer only to certain representatives of *Chelonia*, *Ophidia* and *Lacertilia*. However, the study of the caryotype of *Reptilia* and its comparison with the caryotype of birds would undoubtedly give an idea of the evolution of the caryotype from *Reptilia* to *Aves* and would serve as additional evidence for the relationship of these two animal groups. Elucidation of this problem appears to us the more interesting, as now there are already grounds to postulate the existence of a great similarity between the two classes of animals with regard to determination of sex.

As shown by the works of Nakamura, Oguma and others, in *Reptilia* studied by them the female sex proved to be the heterogametic one (*XO*), while the male sex proved to be homogametic (*XX*).

The present study was devoted to the chromosomal complexes of the turtles *Emys orbicularis* and *Testudo ibera* which belong to *Chelonia* and are almost unexplored cytologically.

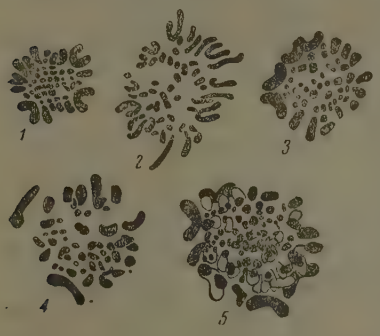
As material for the present investigation have served testes of the turtles *Emys orbicularis* and *Testudo ibera* fixed following Hance, Painter and Minouchi. The best results were obtained from the first and especially the last methods of fixation. The sections were 10—12 μ thick and stained with iron haematoxylin following Heidenhain.

The drawings of equatorial plates were made with eyepieces 10 × and 20× in combination with 1/12 immersion; the length of the tubus was equal to 170 mm. Drawings were made with the aid of Abbé's camera lucida. For chromosome counts only such spermatogonial plates were selected which could not raise any doubt as to the number of components present.

Chromosomes of *Emys orbicularis*. The study of a great number of spermatogonial equatorial plates has clearly shown that chromosomes of *Emys orbicularis*, like in all the other turtles studied, differ in size from each other forming a series with all intergradations from the large V-shaped chromosome to the small rounded «point-like» chromosome (figure, *1*). The point-like chromosomes so far as may be judged from spermatogonial plates are extremely small, so that they are hardly visible and may, therefore, be easily overlooked. These small point-like chromosomes may be convenient-

ly called microchromosomes. They do evidently not correspond to microchromosomes of certain other forms of *Reptilia*, particularly of *Lacertilia*, as in the latter there is a clear difference (without intergradations) in sizes of the large and small chromosomes. It seems to us also convenient to call macrochromosome the larger chromosomes of the complex in question, as it is done, for example, by Nakamura (4).

It should be added that chromosomes of somatic cells (as could be judged from the single plate of a follicular cell found during the study of slides) differ from the spermatogonial chromosomes in being longer and thinner. Owing to this, the small point-like chromosomes are here very well visible and may be easily counted (figure, *2*). In spermatogonia the chromosomes are more condensed and, therefore, are thicker and shorter. It is conceivable, therefore, that somatic cells should be preferred to spermatogonia for the study of chromosome morphology. For comparison a somatic and a spermatogonial plates are presented here.



As shown by chromosome counts in equatorial plates of somatic and spermatogonial cells, the diploid chromosome number in *Emys orbicularis* is equal to 50.

As may be seen from the figure, *2* among the chromosomes of the complex two large V-shaped chromosomes are distinguished; one of them possesses equal arms, while the other consists of unequal arms. Further there are two pairs of long rod-like chromosomes which are pointed on the one end and somewhat thickened on the other. Then follow four pairs which are also rod-like, but not as long as the preceding ones; finally, there are two pairs of small V-shaped chromosomes. Thus, one can distinguish in the complex of *Emys orbicularis* a group of macrochromosomes consisting of 10 pairs; the remaining 15 pairs of chromosomes, beginning with the very small point-like chromosomes up to the comparatively large rounded chromosomes, constitute the microchromosomal group of the complex. The complex studied is this very similar to that of *Sternothaerus odoratus*, and differs from the set of *Clemmys japonica* in containing two additional macrochromosomes which make the diploid chromosome number equal to 52.

However, this difference may be easily explained inasmuch as the two extra rod-like macrochromosomes in *Clemmys* may be ultimately referred to the one V-shaped chromosome of *Emys* or of some other 50-chromosome species.

Chromosomes of *Testudo ibera*. The chromosome complex of *Testudo ibera* does not differ materially from that of *Emys orbicularis*. Spermatogonial plates in this species look like those of *Emys orbicularis*; the chromosomes are short and thick (figure, *3* and *4*), owing to which fact classification is very difficult. We failed to discover in *Testudo ibera* a somatic plate necessary for establishing of chromosome morphology.

Nevertheless, by a close inspection of the equatorial plate represented in the figure, *4* all the macrochromosomes may be, to a certain extent, arbitrarily divided into 4 types, as it was made in the case of *Emys orbicularis*. One pair of large chromosomes is detected first; judging by the bending of its components, one should take it for the V-shaped pair with equal arms. The second pair of V-shaped and somewhat smaller chromosomes also corresponds to a similar pair of chromosomes with unequal arms, since in the

one of the components of this pair the second shorter arm is clearly visible. Then follow four thick oval rod-like chromosomes which evidently correspond to the four long rod-like chromosomes from somatic cells. The following eight comparatively thick but smaller oval chromosomes correspond to the eight small rod-like chromosomes of the somatic set; finally, there are four somewhat thinner V-shaped bent chromosomes. It should be noted that in the latter case, with the only exception, the bending of chromosomes is very slightly expressed.

In *Testudo ibera*, like in all the other 50-chromosome forms of turtles, there is thus a group of macrochromosomes consisting of 20 elements. The remaining 30 microchromosomes constitute the other group of the complex; they are characterized by a point-like rounded shape and display a series of intergradations with regard to size. In another plate (figure, *3*) this seriation of chromosomes with regard to shape and size is somewhat less expressed.

Inspection of a sufficient number of spermatogonial plates has shown that the diploid chromosome number in *Testudo ibera* is equal to 50.

In the species in question, in contrast to the other turtles which were studied cytologically, we succeeded in discovering a tetraploid plate (figure, *5*). Unfortunately, this plate was the only one and, despite its comparative clearness, we were able to count the chromosome number only approximately. We have found here 98 elements, a number which is very close to the expected tetraploid number, i. e. 100. One should attribute the deviation to compactness of the chromosomes and to their superposition on each other in certain parts of the plate.

As seen from the figure, one may without great difficulty count in the plate as many as 40 large chromosomes which correspond to the macrochromosome group of a tetraploid; all the remaining chromosomes which vary in size from point-like to comparatively large rounded ones, constitute the group of microchromosomes.

In the figure there are also clearly visible 6 large V-shaped chromosomes, instead of 8; this discrepancy should probably be explained by unfavourable position of the other V-shaped chromosomes.

In the two representatives of turtles studied by us the chromosomes are, as a rule, arranged at the periphery of the plate, while the microchromosomes are located at its centre, i. e. they are characterized by the same chromosome arrangement which is typical of *Reptilia* in general.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

GENETICS

EXPERIMENTS ON SELECTION FOR FITNESS

By M. M. KAMSHILOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 11.II.1939)

Natural selection is selection for adaptation to concrete life conditions. Fitness of the organism to the surrounding medium is a character of an extreme complexity. It depends on such qualities as viability, fertility, not to speak of smaller details of organization. To say that natural selection is selection for fitness means to say that it is selection for the sum total of characters of the organism.

Practical breeding as well as numerous experimental studies on selection for maximum manifestation of certain individual characters witness to efficiency of such an one-sided selection. Experiments on direct selection for fitness are nearly lacking.

There are, it is true, numerous indirect indications in favour of efficiency of such a selection under experimental conditions. Here may be classified, first, experiments on one-sided selection. In such experiments a strong decrease of viability and fertility is observed as a rule. A conclusion may be made that the initial line was heterogenic with respect to hereditary factors of different viability and fertility, and that selection differently directed might select genotypes determining higher viability and fertility. Secondly, existence of mutations leading to lethal or semi-lethal effect and to various grades of sterility speaks for their unequal value with respect to fitness to the given concrete conditions.

Dobzhansky and Queal (1) have shown that a wild population of *Drosophila melanogaster* is heterogeneous not only with regard to mutations which decrease the average viability of a line, but with respect to mutations of the opposite kind also.

All these data make selection for increased fitness on principle practicable. We are justified in expecting the possibility of shifting this character under experimental conditions too.

As object for the investigation was taken a heterogeneous population of *Drosophila melanogaster*. Fitness was expressed in the number of flies hatching within the first three days from the eggs deposited by a fertilized female. It is evident that in this index were included such basic characters of prime importance for fitness as fertility, rate of development and viability of the developing flies. The method of the experiments was as follows.

Thirty bottles of individual cultures were started. On the third day from the beginning of hatching the number of flies hatched was counted in each

bottle; the next generation was started in a form of individual culture from three bottles which produced maximum numbers of flies. In order to avoid deleterious effect of close inbreeding, the flies from the three bottles with maximum numbers of individuals were mixed together prior to start new generation. Four experimental series falling into two groups were conducted.

The first group—normal flies. They came from crossing females of the Berlin population with males of the Crimea line.

The second group—*eyeless*. This culture was also highly heterogeneous in genetical sense, owing to crossing with flies from the Crimea and Berlin lines. The line was homozygous for the gene *eyeless*. Each group of experiments consisted of two series, the experimental (*o*) and the control (*k*) ones. In the experimental series the flies from the three bottles which gave the maximum numbers of progeny were subjected to the influence of a temperature of 0°C during 24 hours prior to starting the new generation. In control series this was not done. The next generation was started simultaneously from the control series and from the experimental one. Altogether selection was continued during 22 generations within the «normal» group and for 20 generations within the *eyeless* group.

The results of the experiments are represented in Tables 1 and 2 and in Figs. 1 and 2.

Table 1

Normals

Generations	Control				Experiment			
	$M \pm m$	σ	$C\%$	n	$M \pm m$	σ	$C\%$	n
1—5	21.98±1.38	17.4	79.2	32.0	26.3±1.62	18.82	71.7	27.0
6—11	39.35±1.37	17.8	45.3	27.8	36.2±1.57	18.8	52.0	23.7
12—16	39.75±1.14	13.75	34.6	28.8	25.5±1.37	11.4	44.6	13.8
17—22	57±1.29	16.7	29.3	27.8	26.2±1.56	14.2	54.2	13.7

Table 2

Eyeless

Generations	Control				Experiment			
	$M \pm m$	σ	$C\%$	n	$M \pm m$	σ	$C\%$	n
1—5	11.97±0.9	9.75	81.5	23.0	16.3±0.81	7.05	43.2	15.2
6—10	25. 2±1.34	12.7	50.4	25.0	25.6±1.54	13.2	50.0	14.6
11—15	24. 5±0.83	8.86	36.1	22.8	23.4±1.17	10.7	45.7	16.6
16—20	35±1.09	12.7	36.3	27.2	22.6±1.6	12.35	54.6	14.0

The tables represent the changes during the process of selection of the mean number of flies per bottle (M), of the variation in this number (σ), of the coefficient of variation ($C\%$) and of the mean number of bottles per generation (n). Data are arranged not by individual generations, but summarized for five or six generations (see the column «generation»). This makes it possible to eliminate accidental fluctuation of the selected character from generation to generation.

Fig. 1 represents the variation of the mean number of flies per bottle in the control and in the experimental series of the «normal» group. Fig. 2 shows, respectively, the same for the group *eyeless*. Solid line in the two cases shows the control, dotted line represents the experiment. The results obtained make it possible to draw the following conclusions:

1. There is a certain synchrony in fluctuation of the number of flies per bottle in each generation of experimental and control series (Figs. 1 and 2). The synchrony is greater in the first generations and almost disappears in the last generations. This demonstrates that flies which develop from gametes subjected to cold react to varying enviromental conditions (in the first instance, quality of food, for temperature was kept throughout at 25°C) in a similar manner as those from gametes not treated by cold. Along with differentiation of the genotypes, as a consequence of selection, the similarity of reaction disappears. Similar genotypes react in a similar manner to fluctuation of environmental conditions, while the different ones react differently.

2. Selection for maximum number of flies per bottle increases the mean number of flies per bottle in the control series. The mean for the first five generations of the normal line was 21.98 ± 1.38, and for the last six genera-

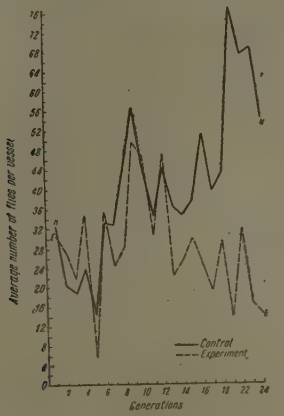

Fig. 1.

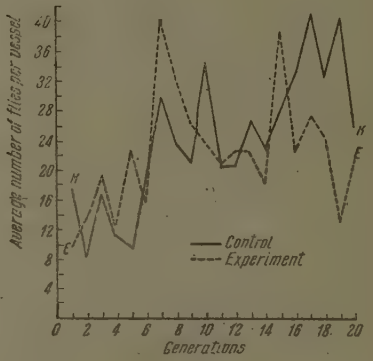

Fig. 2.

tions—57 ± 1.29 (Table 1). The number has increased more than twice. The mean for the first five generations of the line *eyeless* is 11.97 ± 0.9 and that for the last days—35 ± 1.09 (Table 2). The number has increased almost three times. The result obtained in the control series of the two groups shows that under the conditions of the laboratory experiment one may greatly increase the number of flies per bottle which reveals the better fitness of the given line to conditions of development.

3. The value of the absolute variability (σ) is not changed by the process of selection in the control series. This leads to a strong decrease of the coefficient of variation ($C\%$). In other words, the relative variability of the fitness under experimental conditions decreases. This speaks for an increase of homozygoty of the lines which were subjected to selection. An increase of fitness thus takes place at the expense of a certain decrease of heterogeneity. These data demonstrate that the high heterogeneity and variation which arises from it not always mean high fitness. The main thing here is the character of heterozygoty ([3]) and, besides, the concrete environmental conditions. This result is in accord with the ideas developed by Haldane ([2]) with regard to interrelations between fitness and variation.

4. No increase of viability comparable to that occurring in controls is observed in experimental series of the two groups.

In the group *eyeless* certain increase of the number of flies per bottle is observed during the first 10 generations, it remains further unchanged (or is rather somewhat reduced) (Table 2). In the «normal» group there is again

a slight increase of the character selected for during the first 10 generations (Table 1). Then its value drops without reaching the initial one (first generation, 26.3 ± 1.62, the last ones—26.2 ± 1.56).

In other words, the treatment of adult flies with a temperature of 0° during the 24 hours almost completely annihilates the effect of selection upon greater fitness to breeding conditions.

The analysis of the results shows, however, that the effect of temperature is an indirect, rather than a direct one. Let us compare the mean numbers of flies per bottle (M) in the first five generations of the control and experimental series in the two groups. No decrease of the number of flies per bottle is observed in the experimental series, rather an opposite change is observed.

Obviously the treatment with cold by itself is not a factor decreasing viability of gametes and, therefore, not a factor affecting fertility. Neither does it decrease viability of zygotes. A strong decrease of the mean number of successful cultures (n) takes place at the same time in two groups as a result of a high proportion of mortality among parents treated with cold.

In other words, treatment with cold which causes great mortality among adult flies decreases the proportion of successful cultures. The decrease of the number of successful cultures diminishes the efficiency of selection. The decrease of intensity of selection for fitness leads to disorderly homozygotization. Disorderly homozygotization, as a rule, leads to decline of viability and fertility.

5. Non-effectiveness of selection in the experimental series thus points to the importance of the scale of the experiment. Obviously experiments dealing with material of the order of 15 bottles for the culture *eyeless* and 19.5 bottles for the «normal» cultures are of too limited a scale for producing shift. On the other hand, in experiments conducted on a larger scale (over 30 bottles per generation) there are all grounds to expect a still greater progress of selection.

6. A comparison of results obtained in experimental and control series shows that increase of the number of flies per bottle in the control series cannot be explained by progressive improvement of some uncontrollable conditions of development (variation in air humidity, variation in the quality of individual ingredients of food, etc.). Experimental series were all the time conducted parallel to the control ones under identical conditions, and no increase in the character selected for was observed in them.

Conclusions

Under laboratory experimental conditions a rather significant increase in the fitness of a population of *Drosophila melanogaster* to conditions of development may be attained through selection; the fitness is expressed in an increase of the mean number of developed progeny per one pair of parents. The effectiveness of selection to a large extent depends on the scale of experiment.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PLANT PHYSIOLOGY

EFFECT OF IONS OF ALUMINIUM AND PHOSPHORIC ACID ON BIOLOGICAL PROPERTIES OF PLANT PROTOPLASM

By L. I. SERGEYEV and K. A. SERGEYEVA

(Communicated by A. E. Fersman, Member of the Academy, 1.II.1939)

The toxicity of aluminium ion, which is in excess in the soil of subtropical and tropical countries, has been reported in a review by Prof. Potapov (2) and in other works, belonging chiefly to English and American authors. In this country such soils occur in Transcaucasus, where most valuable crops, as tea, citrus trees, tung and others are cultivated.

Experiments have shown that the toxic effect of aluminium ion can sometimes be suppressed by applying ultra-doses of superphosphate or some other phosphate fertilizers. Most representative in that respect are the experiments of Potapov (3), who has been successful in increasing the yield of various crops, tea for instance, ten and more times.

In connexion with these experiments considerable interest is furnished by the question of how the ions of aluminium and the ions of *o*-phosphoric acid act mutually and how they act on the plant. To clear up that question, one of the authors of the present article started an investigation in the laboratory of prof. Potapov, Kinelsk Agricultural Institute (1933).

From the very start we were compelled to give up the supposition that active aluminium* might be precipitated by the solutions of *o*-phosphoric acid, since no precipitate would appear in the mixture of these solutions. No more truth was in the supposition that some physico-chemical factors might reduce the activity of aluminium ion considerably. Measuring the specific conductivity of the mixture $Al_2(SO_4)_3+H_3PO_4$, we found it nearly to equal the sum of ion conductivities stated for each of the components separately.

There remained, therefore, only one possible assumption—that it is by dint of physiologically antagonistic relation between aluminium ion and phosphoric acid that the toxicity of the former is suppressed by the latter. Corroborative evidence of this assumption has been obtained by our experiments. We made the seed of cultivated cereals germinate and grow in Buchinger's (1) apparatus, and the size of the above-ground parts of the seedlings, the storage of dry matter, as well as the number of seeds germinated, testified the fact of physiological antagonism between the ions in question. With a mixture of aluminium salt and phosphoric acid solutions

* Aluminium was taken for experiments in the form of sulphate and chloride solutions.

all these indices showed values which are intermediate, as compared to those obtained with each of the above solutions separately ([4], [5]). In the present article we adduce nothing but a photograph to demonstrate the antagonistic relation between the ions of aluminium and *o*-phosphoric acid (Fig. 1).

We supposed thus antagonism to consist in the different nature of the action the ions concerned exert on the biological properties of the cell protoplasm in a vegetable organism. Below we refer to some data obtained in the years 1936 and 1937.

Fig. 1. 10 day old seedlings of summer wheat 341. 1) seedling from solution of aluminium sulphate, 2) from «mixture»[$Al_2(SO_4)_3+H_3PO_4$] and 3) from solution of *o*-phosphoric acid.

1. Dynamics of Protoplasm Viscosity. The method of estimating the viscosity of protoplasm from the time and form of plasmolysis, devised by Cholodnyj([9]) and Weber ([7]), made it possible to ascertain the nature of the influence produced by the ions in question upon the inner friction of the protoplasm. For that purpose slices of epiderm from fleshy scales of an anthocyanine-bearing onion were kept in the solutions of aluminium salt (0.001 to 0.01 mol.) and *o*-phosphoric acid (pH = 2.8—3.2) for 24 hours. Control, distilled water (pH = = 5.8 — 6.0). From the form of plasmolysis * which predominated in the slices we had to conclude that the aluminium ion increases the viscosity of the protoplasm (concave and even convulsive form), while *o*-phosphoric acid acts contrariwise (convex form). The figures annexed bears witness of this relation by showing the predominating form of plasmolysis within a period of 20—30 seconds after the outset of plasmolysis (Figs. 2, 3 and 4).

An increase of the viscosity of protoplasm due to the influence of aluminium ion was recorded also by other authors, Szücs ([8]) for instance.

2. Dynamics of Protoplasm Permeability. The permeability of the protoplasm proved also to be liable to the opposite effects of the ions tested. In this experiment we resorted to the method of electrometric estimation of electrolytes passed from the living tissues of the plant into distilled water by way of exosmosis. The above-ground parts

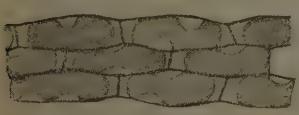

Fig. 2. Section from aqua destillata.

Fig. 3. Section from solution of aluminium sulphate.

Fig. 4. Section from solution of *o*-phosphoric acid.

of wheat seedlings, which while growing were given the solutions of aluminium salt and *o*-phosphoric acid respectively (control, distilled water), were, in equal amounts by weight, immersed into equal amounts of distilled water in the test tubes (lixiviated by water vapour) and kept there at 10°C for 24 hours. The indices of specific conductivity are referred to in Table 1.

As seen from Table 1, the greatest value of the index applied is with the seedlings given *o*-phosphoric acid and the least is with those which received aluminium sulphate. The values obtained with the mixture and distilled water are intermediate.

* Indicator, 1.0 mol. of sucrose, as well as 0.5—1.0 mol. of NaCl or Na_2SO_4.

Table 1

Specific Conductivity ($K \times 10^{-10}$) of Distilled Water after Exosmosis of Electrolytes from 10 Day Old Wheat Seedlings (average from 3 measurements)

Experimental variant	Specific conductivity
Control (distilled water, pH=6.2)	454
0.005 mol. of $Al_2(SO_4)_3$ (pH=3.9)	327
Solution of *o*-phosphoric acid (pH=5.3)	727
Mixture $Al_2(SO_4)_3 + H_3PO_4$ (pH=3.9)	631

It may, therefore, be concluded that the permeability of the protoplasm goes up under the influence of *o*-phosphoric acid, while it goes down under the influence of aluminium ion.

3. Dynamics of Water Retaining Capacity. The changes in the viscosity and permeability of the protoplasm are bound up intimately with the dynamic of colloid-chemical processes ([6]). This statement is borne out by our data on water retaining capacity and on water content in the living tissues of the same plants (Table 2 and Fig. 5).

Table 2

Content of Water in Living Tissues of Winter Wheat Seedlings] in % of Total Weight (average from 4 measurements)

Experimental variant	Water in % of total weight
Control (distilled water, pH=6.1)	88.5
Aluminium sulphate solution (pH=3.8)	85.3
o-phosphoric acid solution (pH=2.8)	89.6

From Table 2 and Fig. 5 it is obvious that the increase of viscosity and the decrease of permeability of the protoplasm due to aluminium ion went parallel with the decrease of water ratio in the tissues and the increase of their water retaining capacity. On the contrary, *o*-phosphoric acid caused the ratio of water to go up in the tissues of the seedlings and reduced their water retaining power.

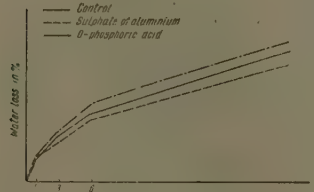

Fig. 5. Water loss to total water content in seedlings.

Furthermore, data have been obtained on the change of the sign or value of the electric charge in the cellular juice. These changes have been stated in special experiments on electrophoresis. But the data at our disposal cannot be reported in the present article on account of the space available.

The conclusion to be drawn from what has been said here is that the antagonism between aluminium ion and *o*-phosphoric acid consists in the diametrically opposite action of these ions upon the biological properties of plant's protoplasm: viscosity, permeability, water retaining power and others. But it is by the dynamics of these properties that the intensity of physiological processes in plant and its resistance to dehydrating influence of environment is determined.

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PLANT PHYSIOLOGY

IONIC ACTION AS A MEANS OF CONTROLLING RESISTANCE AND GROWTH OF PLANTS

By L. I. SERGEYEV and K. A. SERGEYEVA

(Communicated by A. E. Fersman, Member of the Academy, 1.II.1937)

In the preceding article ([8]) we have shown that the ions of aluminium and those of *o*-phosphoric acid are diametrically opposite to each other as regards their influence on biological properties of plant's protoplasm. But the dynamics of these properties is a factor in the intensity of physiological processes and in the resistance of plants to dehydration (whether by drought, or frost, or salt excess in soil). It has been shown, for instance by Golush ([2]), that the process of hardening, as well as the forms resistant to frost, are characterized by an increase in the permeability of the protoplasm.

Such increase has been stated by Filippenko ([11]), Shestakov and Sergeyev ([12]) in winter cereals on their going through the first two stages of development established by Lyssenko. On the other hand, a sharp drop of frost resistance is known to happen at these stages.

An increase in viscosity of the protoplasm has been recorded by Kessler ([6]) and Bělehradek ([1]) in the case of plants subjected to hardening or passing winter in dormant condition.

According to a great many data referred to by Maximov ([7]), the amount of water in and the water retaining power of the protoplasm are potent factors in frost and drought resistance of plants.

From all these works it was to be supposed that the resistance of plants to frost should increase under the action of aluminium ion, while a decrease of this characteristic should be expected from the effect of *o*-phosphoric acid. Corroborative evidence of this supposition has been obtained from a number of experiments on freezing 10 day old seedlings of various wheat varieties from seeds grown in the above solutions (control, distilled water). The data of one of those experiments are referred to in Table 1.

From Table 1 it may be seen that frost resistance of seedlings is increased by solutions of aluminium sulphate, while it is reduced by *o*-phosphoric acid. The ratio of survived controls shows intermediate values.

Quite opposite is the picture observed on estimating the intensity of physiological processes. In his experiments on various plants Stoklasa ([10]) has revealed an inhibition of seed germination by the solutions of aluminium salts. On the other hand, Gračanin ([3]), experimenting with seeds of various crops, has found that process to be stimulated by *o*-phosphoric acid. These statements fall in line with the results of our experiments, since we also

Table 1

Ratio (in %) of 10 Day Old Seedlings Survived after Freezing at 5°C for 25 minutes

Cereals tested	Control (distilled water, pH=6.4)	Aluminium sulphate (pH=3.6)	Aluminium sulphate (pH=3.4)	*o*-phosphoric acid (pH=3.6)	*o*-phosphoric acid (pH=3.8)
Rye Elisseyevskaya	80	100	60	0	0
Winter wheat 329	40	90	80	0	0
Rye wheat hybrid 434/154	80	100	100	40	20
Winter wheat 237	50	100	100	60	20
» » A2453	80	90	80	40	20
» » «Ukrainka»	40	70	100	20	0
» » «Kooperatorka»	30	80	60	10	0
Spring wheat 62	40	70	100	0	0
Average	55.0	87.5	85.0	21.3	7.5

stated an increase in the germination energy of different cultivated cereals under the influence of weak concentrations of *o*-phosphoric acid and at the same time observed a depression when germination was taking place in solutions of aluminium sulphate.

In a paper by Znamensky ([4]) the decrease of transpiration intensity due to the influence of aluminium ion is reported, and in another paper by Ivanov ([5]) data are referred to, which show this intensity to increase in the case of *o*-phosphoric acid.

Naturally enough, the intensity of growth will be also dependent on the presence of one or the other of the above ions and therefore variable within a rather wide range. The data presented in Table 2 are obtained from measuring the above-ground parts of seedlings of various cultivated cereals given different solutions.

Table 2

Size of Above-Ground Parts of 10 Day Old Seedlings in cm (average from 10 measurements)

Cereals tested	Control (distilled water, pH=5.8)	Aluminium sulphate (pH=3.6)	Mixture $Al_2(SO_4)_3 + H_3PO_4$ (pH=3.0)	*o*-phosphoric acid (pH=3.6)
Winter rye Elisseyevskaya	7.05	5.23	4.53	8.07
» wheat 329	5.26	3.06	4.54	9.11
» » 237	5.65	4.07	4.64	8.32
» » «Kooperatorka»	6.02	3.28	4.35	4.54
Spring wheat 62	7.35	5.30	5.55	9.73
» » 432	5.42	2.20	2.74	3.72

The conclusion to be drawn from Table 2 is that in all but exceptional cases the energy of growth process is enhanced by *o*-phosphoric acid, while aluminium sulphate, on the contrary, is a powerful inhibitor of their development. The data obtained with distilled water and mixture furnish evidence of their intermediate influence on growth. The power of aluminium ion to inhibit growth becomes apparent with especial clearness in the

development of roots. Their growth is exceptionally weak in the solutions of aluminium sulphate.

The stimulation of growth by weak concentrations of *o*-phosphoric acid can be made use of to accelerate the regeneration process on vegetative reproduction of plants. Our experiments with root pieces from *Mulgedium tataricum* and twigs from *Populus niger* have shown that the effect of the solutions under test is greatly to further the development of new shoots and the opening of dormant buds. But neither the buds did open nor the shoots developed and even finished with dying when the subjects of experiment were treated with ammonium sulphate, whose effect has thus proved to be in sharp contrast with that of *o*-phosphoric acid.

We are therefore entitled to say that aluminium ion is a promoter of plant's resistance and an inhibitor of various physiological processes, while *o*-phosphoric acid favours the latter, but makes the plant less resistant.

The entire set of experiments on ions of aluminium and *o*-phosphoric acid has once more shown that the condition of a vegetable organism, when it is most resistant to dehydration, is dialectically opposed to the condition, when physiological processes are running with especial energy in it. And it is only through a decrease in the rate of its exchange with environment that the vegetable organism becomes resistant to unfavourable influences.

This regularity, which was treated in more detail in one of the items of a dissertation by Sergeyev (9), is most pronounced in lower and low organized plants being known under the name of anabiosis (13). In higher plants a partial anabiosis becomes apparent during early stages of ontogenesis. The chief forms, in which it is manifested there, are «hardening» and «winter rest».

The control of hardening and growth processes in plants can be based upon the theory of anabiosis as a general biological theory of plant's resistance to dehydrating environmental factors. Multivalent ions (of aluminium and *o*-phosphoric acid) present one of the means which can be turned to account for that practical purpose.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PLANT PHYSIOLOGY

RÔLE OF LEAVES IN THE STORAGE OF STARCH BY POTATO TUBERS AS DETERMINED BY THEIR RANK ON THE PLANT

By V. P. NILOVA

(Communicated by N. I. Vavilov, Member of the Academy, 10.II.1939)

The possibility of directing plant's life activity towards an increase of its practically valuable qualities is not to be achieved without a profound study of its physiological and biochemical properties and, in the first instance, of the relationship between enzymatic system and metabolism. The existence of such a relationship has been recorded by a number of workers (Motes, Oparin and oth.). By making use of the method of vacuum infiltration devised by Kursanov, some of the authors have established that there is a definite ratio between the hydrolytic action of the invertase and its synthetic action. Oparin and Rubin, for instance, have found the ratio between those functions of the invertase to determine the sugar content of several vegetables (beet, onion, cabbage), both quantitatively and qualitatively. This ratio can shift towards one or the other side in response to some external factors, yet it is essentially determined by the species and the variety of the plant concerned.

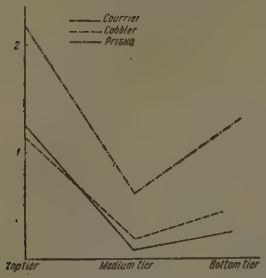
Rubin believes it to account for the plant being more or less capable of early ripening, preservation and so on, and that the regularities the plant reveals during vegetation hold true when it is being stored.

The unpublished data obtained by Nilov and associates on a number of crops have furnished evidence that the ratio between the hydrolytic and the synthetic action of the invertase varies in the leaves according to their position along the stem. By removing leaves from a certain height the authors have been successful in increasing the sugar content in peach fruits. Naturally enough, the process of starch storage in potato tubers should also be related to the physiological function of the leaves, to the process of formation and conversion of sugars in the plant.

Since that question seems to be of importance for plant breeding work in this country and furnishes, moreover, a theoretical interest, we have deemed it wise to start experiments in this direction. The experiment was run with three varieties of *Solanum tuberosum* different with regard to their time of ripening. Courrier and Cobbler—early varieties, and Priska—a late one.

The plants were grown under equal conditions in a levelled ground of the state farm «Krasnyi Pachar», near Leningrad. At the outset of full flowering the leaves taken from different heights were tested for the relation between the hydrolytic and the synthetic action of the invertase. Similar

observations were performed on the species *Solanum Rybinii* and *Solanum andigenum* f. *compiss*. The number of leaves on the main shoot were counted. At the start of the experiment there was usually 9—12 leaves, because the lower leaves were, as a rule, broken by earthing and had already died away at that time. In counting the leaves we went downwards from the floriferous shoot. The fifth, sixth and seventh leaves were referred to the «middle level», the leaves situated upwards and downwards from those—to the «upper level» and «lower level» respectively. Special attention was paid to the fifth and the sixth leaves, since their shape and outline are considered by potato breeders as most distinct varietal characters. We took main shoots from 3 plants for the test, first dividing the leaves according to levels and then dividing them with reference to the lobes. Each lobe was cut into halves along the petiole and the mean sample was tested, due care being taken that the respective lobes from different levels were represented by equal numbers in each of the experimental variants. Recurring to the method of vacuum infiltration, we investigated the trend of invertase activity. In order to estimate the synthetic action of the invertase, the leaves were infiltrated with 0.2 mole solution of equimolecular amounts of fructose and glucose, while 0.1 mole solution of sucrose was used for the estimatoin of the hydrolytic action. The carbohydrates were estimated according to Bertrand before and after hydrolysis.



Ratio between synthesis and saccharose hydrolysis according to the rank of leaves in the *Solanum tuberosum*.

From the data obtained we had to conclude that in each of the three varieties tested of *Solanum tuberosum*, no matter whether they ripen early or late, the ratio of hydrolysis to synthesis varies with the same regularity. The hydrolytic action of the invertase is always predominant in the middleranked leaves, while those of the upper level are characterized by a predominance of the synthetic activity. The respective difference between the levels exceeds far beyond that which is to be noticed, when the same levels in different varieties are compared. The same is true of other potato species.

Table 1

Invertase Activity in Living Tissue of Leaves from Various Potato Varieties (in mg of sugars per 1 g of dry matter per hour)

Variety	Level	Synthesis	Hydrolysis	Ratio of synthesis to hydrolysis
Courrier (early) .	Upper	14.50	11.40	1.27
» » . .	Middle	1.67	22.44	0.074
» » . .	Lower	3.97	15.25	0.26
Cobbler (early) .	Upper	11.12	9.70	1.14
» » . .	Middle	2.96	13.63	0.21
» » . .	Lower	8.33	16.40	0.50
Priska (late) . . .	Upper	30.80	14.10	2.18
» » . . .	Middle	13.50	21.30	0.63
» » . . .	Lower	22.40	17.10	1.31

It would appear that a varying physiological function is performed by the leaves, according to their position along the height of the stem, and that the trend of some fermentative processes (the activity of the invertase in the case considered) is virtually determined by this position rather than by the fact of varietal, specific or even generic origin of the plant concerned.

After having established the trend of this process in leaves from various heights, we set up an experiment with the aim to ascertain the effect the removal of leaves from a given height exerts upon the balance of carbohydrates in potato tubers.

Table 2

Invertase Activity in Living Tissues of Leaves from Various Potato Species (in mg of sugars per 1 g dry matter per hour)

Species tested	Level	Synthesis	Hydrolysis	Ratio of synthesis to hydrolysis
Solanum Rybinii . . .	Upper	20.4	13.37	1.6
» » . . .	Middle	1.43	14.42	0.1
» » . . .	Lower	4.55	11.76	0.38
S. andigenum f. *compiss.*	Upper	9.72	8.95	1.08
» » » »	Middle	2.36	17.79	0.13
» » » »	Lower	2.62	11.57	0.22

Three varieties of *Solanum tuberosum* were subjected to the test: two early varieties—Cobbler and Eihenheimer and a late one—Priska, 10 plants being taken from each of them. 5 plants were stripped of their middle-height leaves, i. e. of the 5th, 6th and 7th leave from above, which were removed at the time of flowering from all the shoots, while on the other 5 plants it was the leaves of the upper and lower level which were taken away.

The average number and weight of the leaves removed from each of the shoots is given in Table 3.

Table 3

Level	Number of leaves removed	Average green weight of one leaf in g	Average weight of leaves per shoot in g	
Upper . . .	4	0.55	2.20	4.2
Lower . . .	2	1.00	2.00	
Middle . .	3	0.82	2.46	

Table 4

Average Weight of Potato Tubers per Plant

Experimental variants	Average weight per plant in g
Control	590
Upper and lower height leaves removed	587
Middle height leaves removed	594

Six weeks later the plants were harvested and the yield of each of them was weighed and analyzed. As seen from Table 4, the total weight of the yield proved not to be affected by the removal of leaves.

The controls which were not deprived of their leaves were grown on the same plot with the experimental plants. The sugars were estimated according to Bertrand before and after hydrolysis; the starch was subsequently converted into sugars by means of the diastase.

Thus, removing the upper and lower leaves we removed nearly twice their number, compared to the first variant, without affecting the normal life activity of the plant. Through creating conditions which favoured the synthetic or hydrolytic action of the invertase, we respectively obtained quite definite results with regard to the storage of starch in potato tubers.

The conclusion to be drawn from the experiment is that the photosynthetic activity in the leaves of the middle level is high enough to provide the plant with sugars, of which the starch is built up.

While favouring the hydrolytic activity of the enzyme in question, we further the migration of plastic materials from the leaf to the tuber. This is the reason to account for the fact that the tubers of the II variant were much more heavily stored with starch as compared to those of the I variant.

Table 5

Content of Sugars and Starch in % of Raw Weight of Tubers

Experimental variant	Without middle leaves				Without upper and lower leaves				Controls (no leaves removed)		
Variety	No of plant	Monosaccharides	Disaccharides	Starch	No of plant	Monosaccharides	Disaccharides	Starch	Monosaccharides	Disaccharides	Starch
Cobbler . .	1	0.07	0.26	12.67	6	0.03	0.29	13.75	—	—	—
» . .	2	0.03	0.26	11.10	7	0.11	0.15	14.70	—	—	—
» . .	3	0.20	0.43	12.00	8	0.10	0.12	17.00	—	—	—
» . .	4	0.04	0.25	10.50	9	0.06	0.09	17.25	—	—	—
» . .	5	0.03	0.30	11.86	10	0.11	0.22	13.45	—	—	—
Average . .	—	0.07	0.31	11.53	—	0.08	0.17	15.63	0.16	0.23	14.50
Priska . . .	1	0.23	0.24	11.32	6	0.28	0.09	19.00	—	—	—
» . . .	2	0.20	0.22	11.92	7	0.25	0.18	14.33	—	—	—
» . . .	3	0.21	0.20	12.50	8	0.24	0.28	14.50	—	—	—
» . . .	4	0.33	0.19	11.55	9	0.23	0.12	14.50	—	—	—
» . . .	5	0.26	0.22	12.16	10	0.33	0.28	14.65	—	—	—
Average . .	—	0.24	0.21	11.89	—	0.27	0.19	15.39	0.16	0.23	13.50
Eihenheimer	1	0.07	0.38	15.00	7	0.25	0.29	16.12	—	—	—
»	4	0.19	0.19	14.05	9	0.20	0.25	16.17	—	—	—
Average . .	—	0.13	0.28	14.52	—	0.22	0.27	16.14	0.06	0.30	15.3

Cobbler +4.1%, Priska +3.5%, Eihenheimer +1.6%. The same is to be noticed when comparing the tubers of the II variant with the controls. Opposite effect is produced by weakening the hydrolytic activity of the invertase in the green mass, and therefore the content of disaccharides was only slightly increased, while that of starch was strongly reduced in the control tubers.

The results obtained permit us to expect with still more confidence that the study of biochemical and physiological properties of the leaves with reference to their elevation on the plant will reveal quite new and very promising ways for controlling the biosynthesis in plants.

Conclusions: 1. The removal of upper and lower leaves from the shoots of potato plant causes the tubers greatly to increase in starch, especially compared to the plants stripped of their middle leaves.

2. To whatever species or variety may the potato plant belong, the hydrolytic activity of the invertase in its middle leaves always predominates over the synthetic activity. This phenomenon may be brought in connexion with the special rôle of these leaves in the storage of starch by potato tubers.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

PLANT ANATOMY

FUNCTIONAL SIGNIFICANCE OF EPIBLAST IN WHEAT EMBRYO

By M. S. JAKOVLEV

(Communicated by N. I. Vavilov, Member of the Academy, 5.II.1939)

The embryo of cereals, wheat in particular, presents a strongly differentiated young organism of a new plant which, some definite conditions being available, enters the cycle of its individual life. Naturally enough, it is of exceptional interest to know the particulars of its structure, the more so that on germination of seed it is in the growing point of the embryo that the qualitative changes characterizing the yarovization stage are taking place.

Morphologically the embryo of wheat is represented by the scutellum, the epiblast, the bud primordium and the seminal radicals. The bud primordium is enclosed in the coleoptile with the first leaflets and the growing point, while the primary radicle is enclosed in a root sheath, the coleorhiza.

There is a large accumulation of literature concerned with the study of the wheat embryo ([1–5]). Most of the experimental efforts have centered about the problems connected with the homology of the principal parts of the embryo.

The epiblast has been variously interpreted as a ligule or a rudimentary organ homologous to the second cotyledon of dicotyledonous plants. As rudimentary it was not credited to have a great morphological importance. The investigators seemed to believe that this rudimentary cotyledon has entirely lost its morphological and physiological significance in the life of the embryo. This is the only reason to account for the fact that neither the authors of taxonomic monographs ([6–8]) nor the physiologists dealing with the questions of germination of cereal seed ([9]) have paid due attention to this organ.

While studying the ontogeny of wheats, we made an attempt to ascertain the rôle of the epiblast and to reveal its structural peculiarities against a background of ecologically different species and varieties.

During its embryonal development the epiblast is somewhat ahead of the scutellum in its differentiation out of the peripheral layers of the embryo. In a ripe embryo it consists of parenchymal cells having thin cellulose walls and large intercellular spaces filled with air. The tissue of the epiblast is rather loose without any hint of differentiation into procambial elements. As to the character of its tissue, the epiblast very much resembles the root sheath (coleorhiza) and lower part of the scutellum.

Its position is immediately above the cotyledonal node which marks the level of the transition from the root to the stem. It is within this compa-

ratively short interval, even if we were to include the first internode, that the connexion between two inversely directed systems is effected. On the one side is the vascular tissue of the seminal root, on the other, the system bound up in its further development with the scutellum, the coleoptile and the first leaflets of the young plant. In the region of the first internode there develop also the first lateral radicles in close proximity to the epiblast. We thus find the epiblast by dint of its position to be associated with the most important and responsible parts of the organism, viz., the first internode and the vascular system of the transitional region—a circumstance disregarded by morphologists and physiologists.

Fig. 1. Outlines of epiblast in different wheats, when examined from above, from the side opposite to the scutellum: I. *Tr. vulgare* v. *ferrugineum* Körn, «Banat» 10912, Hungary. II. *Tr. vulgare* v. *erythrospermum* Körn. 10011, Abkhazia. III. *Tr. vulgare* v. *velutinum* «Thule II» 20326 Sweden. IV. *Tr. turgidum* v. *speciosissimum* Körn. 20643, Spain.

The morphological significance of the epiblast stands most forcibly out when we are comparing ecologically different types of wheat. It is strongly reduced for instance in *Tr. turgidum*, *Tr. Macha* and in some varieties of *Tr. vulgare* (Fig. 1). *Tr. turgidum* is a typical hydrophyte, growing under conditions of high humidity, and so is also *Tr. Macha* which delights in humid subtropical regions of Transcaucasia ([10]). *Tr. vulgare* includes diverse ecological types within its wide area of distribution, and accordingly the size and form of the epiblast in it is highly variable.

Among the wheats with sharply pronounced differences, regarding the development of the epiblast, one may properly refer the winter wheats of Sweden, Holland, Abkhazia, on the one hand, and various Banat varieties, on the other (Fig. 1).

The former originate from the humid countries of Western Europe, the latter from the dry steppes of Hungary and Ukraina.

The figures annexed (Figs. 3 and 4) were taken from epiblasts in wheat embryos of the same variety (*lutescens* Al.), but of different origin. It may be seen that the epiblast is more strongly developed in *lutescens* 062 from Saratov test-station (Fig. 3) than it is in *lutescens* 25157 from Holland (Fig. 4).

The same picture has been revealed by comparing a set of other varieties chosen with reference to the character of drought resistance (specimens picked out by I. A. Stefanovsky). As seen from Fig. 2, the epiblast is much more strongly developed in xeromorphous, drought resistant wheats, such as *graecum* 0289 from Middle Asia (*b*), *erythrospermum* 0841 from Krasny Kut (*d*) and *lutescens* 062 from Saratov. The varieties liable to drought as *lutescens* 28316 from Finland (*c*), *Milturum* 25139 and *ferrugineum* 19138 from Germany (*a*) have their epiblasts weakly developed.

Fig. 2. Outlines of epiblast in different varieties of *Tr. vulgare*: *a*—v. *ferrugineum* Körn. 19138, Germany; *b*—v. *graecum* Körn. 0289, Middle Asia; *c*—v. *lutescens* Al. 28316, Finland; *d*—v. *erythrospermum* Körn. 0841, Krasny Kut.

Such are the facts we met with in studying wheats in their individual development.

In view of the sharp differences revealed in the character of the epiblast in such definite ecologo-morphological types, as *lutescens* from Sweden and Holland, on the one hand, and *lutescens* 0329 from Saratov test-station, on the other, are, we may suppose that the epiblast is a physiologically active

organ in the general system of a wheat embryo. There are reasons for believing it to participate directly in absorption of water at the time of seed germination. This function of the epiblast is favoured by the loose spongy structure of its cells having thin cellulose walls. Water absorbed by the delicate tissue of the epiblast can easily enter the vascular system which develops on the germination of the embryo. Epiblast is a kind of sponge, a condenser of moisture, which is so necessary at the outset of the independent life of the embryo. We are confirmed in this conviction by the experiments of a number of authors ([11–12]) on absorption of water by different portions of a wheat kernel. The result of those experiments is that in an uninjured grain water is absorbed exclusively by the embryo.

It is not a matter of chance that droughty steppes produce cereals whose epiblast is strongly developed, large and broad as in *Stipa pennata* and

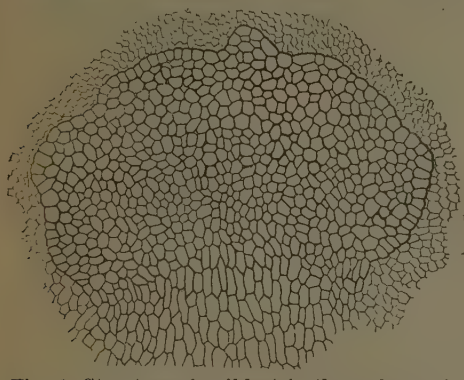

Fig. 3. Structure of epiblast in the embryo of *Tr. vulgare* v. *lutescens* 062, Saratov exper. station.

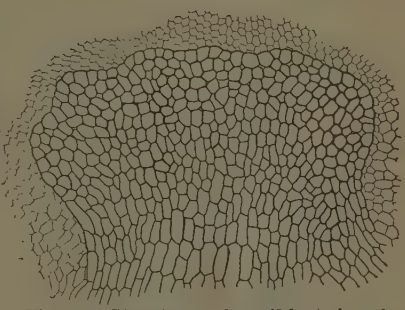

Fig. 4. Structure of epiblast in the embryo of *Tr. vulgare* v. *lutescens* 25157, Holland.

S. viridula. It is worthy of note that with the epiblast being strongly developed, a feature peculiar to the xeromorphous varieties of wheat, there is also observed a well differentiated second pair of lateral seminal roots. Thereby one of the indispensable prerequisites is created for the development of the young seedling under the conditions of soil drought.

The possibility is not excluded that the varieties most resistant to soil drought at the first period of their development will have their epiblast most strongly developed and at the same time will show a well differentiated root system in the embryo.

It will be obvious that further inquiry has to trace the character of the development of the epiblast and seminal roots in various wheat varieties, in order to make possible an adequate use of the particulars in the structure of the embryo for the purpose of selection and seed culture.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

ZOOLOGY

FAMILY *FIDELIIDAE* AND MORPHOLOGICAL CONVERGENCE AMONG BEES *(APOIDEA)*

By V. V. POPOV

(Communicated by S. A. Zernov, Member of the Academy, 15.I.1939)

It is common knowledge that the evolution of the group of bees was proceeding parallel to that of the flowering plants; the development of the one was conditioned by and almost concurrent with the development of the other. Morphological adaptation of flowering plants and bees to one another have been khown and long since recognized as classical. *Apoidea* are morphologically a monotonous group, as the adaptations affected many of the external systems of characters, such as: mouth parts, collecting apparatus, hair clothing, details of venation, structure of limbs, chitinous envelope, etc. The subsequent and non-concomitant parasitism of a number of genera (and groups) from various families has, in its turn, called forth a number of additional morphological (and other) adaptations, which made the morphological differentiation of groups still more involved. A number of characters were greatly effaced, while many others arose independently and evolved convergently. Historically the taxonomy of the group was based, as a rule, on external characters. There were established externally uniform, extensive polytropic genera, sometimes comprising many a thousand species; a number of misleading conceptions have arisen in regard to the natural position of some tribes, genera, etc. in the system of *Apoidea*. The structure of the genital armature was not studied then, whereas characters of genitalia and the adjoining sternites may serve as the most unfailing criterion for determination of the actual position of the group, genus, and their homogeneity. Being endogenous, the system of characters of genitalia was scarcely if at all subjected to the direct influence of adaptation to pollination of flowering plants, or to parasitism, and should therefore a priori reflect more fully natural intrageneric groupings, phylogenetic relationships between genera, family confines, and serve as the main criterion for their determination.

The designation of family *Fideliidae* was suggested by Cockerell (4) for two endemic South African genera *Fidelia* Friese (6) and *Parafidelia* Brauns (2). *Fidelia* comprise at present 8 species, *Parafidelia*—4. Friese (1900), having established the first genus, classed it among the genera *Eucera*—*Anthophora*, i. e. referred it to the family *Anthophoridae* (subfamily *Podaliriinae*, according to Friese's scheme). Cockerell (4) associated *Fideliidae* with the subfamily *Anthidiinae*. Both Friese and Cockerell furnished proof and had good reason for advancing such suggestions. This may be accounted

for by the combination of external characters of the two genera, unusual for *Apoidea*, viz. long, well developed proboscis, two expanded and elongated basal joints of the (lower) labial palps, a long upper lip, well developed abdominal brush, alongside of well developed pollen carrying hairs on the hind legs, Anthidium—similar in form to tergite VII, dilated joints of the front pair of feet in the male; their belonging to the higher bees seemed beyond doubt.

However, the study of the hitherto unknown genital armature in the males *Fidelia villosa* (specimens kindly sent by Dr. Blüthgen and Prof. Cockerell) has shown that the family *Fideliidae* is to be classed, together with the families *Colletidae*, *Diphaglossidae*, *Prosopidae*, and partly *Panurgidae*, among the lower bees. The unusual four-lobed form of tergite VII (Fig. 1, *D*), elongate, well developed, though feebly diverging basal lobes, three pairs of apical lobes, a particularly long, narrow apical lobe fringed

Fig. 1. *Fidelia villosa* Brauns; *A* and *B*—dorsal and lateral aspect of the genitalia; *C*—sternite VII; *D*—sternite VIII.

with long hairs, testify safely to direct affinity between *Fideliidae* and lower bees. Tergite VII of the form indicated above, unknown in higher bees, is rather characteristic of the lower kind (7). The basal lobes bear closest resemblance to some representatives of *Panurgidae* (s. l.). The three-lobed apical part of tergite VII rather inclines to that in the genus *Diphaglossa*, though the latter has one greatly elongated pair of lobes extending straight backwards, parallel to the basal ones. Narrow stipites of the genitalia (Fig. 1, *A* and *B*) with apex as if slightly separated, densely set with hair, with greatly reduced volsellae, and sagittae greatly expanded dorso-ventrally, recall most of all representatives of the family *Colletidae*. This similarity in structure of the genitalia and adjoining sternites to two allied families of lower bees, of which one *(Diphaglossidae)* is even considered a subfamily of the other, is not incidental. The character of the hair clothing, the structure of the apparatus for pollen collection, the length of the proboscis—are characters of a more adaptive nature than the structure of the genital armature.

Among the bees are known cases of occurrence of a long and pointed proboscis amidst the lower short-tongued and obtuse-tongued (obtusilingues) group: *Eupaleorhiza*, *Paleorhiza*, *Macroglossa (Prosopidae)*, *Jomelissa*, *Melittodes* and some species of *Andrena (Andrenidae)*. In all these instances, however, the basal joints of the lower labial palpi retain their primitive form. However, in the family *Panurgidae*, in the modern broad sense, i. e. as

embracing the families *Dufoureidae*, *Macropididae* and *Panurgidae* ([8]), are known genera with a relatively short proboscis and short upper lip without mandibula, as well as with a long proboscis and more or less long upper lip, enclosed by mandibulae *(Camptopoeum*, *Melitturga*, etc.), the basal joints of the labial palpi in *Melitturga* are correlatively acquiring the shape characteristic of higher bees. For this reason *Melitturga* ([3]), as well as *Diphaglossa*, have been for a long time erroneously referred to higher bees *(Anthophoridae)*.

Elements of a vestigial abdominal brush may be observed in a number of species not referable to the higher group of *Gastreligidae*, *Dasypoda*, some of the *Halictus* and others. In *Halictus* the abdominal brush is already used as a «working tool». Brauns ([1]) pointed out that *Fidelia* uses for collecting pollen but its abdominal brush; the hairs of the hind legs take no part in this work. In the females of *F. villosa*, which I have studied, pollen may be seen on the abdominal brush alone. This in a way may be considered as evidence that in comparison to the sweeping brush on the hind leg, the presence of a brush on the abdomen is a new arrangement. My observations on *Andrena fedtschenkoi* F. Mor. have shown that this endemic and strictly oligotropic bee sweeps the pollen from the flowers *Tanacetum unbelliferum* with the hairs of the apical abdominal sternites; probably such method of pollen collection may be reflected in morphological changes—the appearance of a full abdominal brush, quite independently of *Gastreligidae*. *A. fedtschenkoi* is oligotropic to *Tanacetum*—representative of the family *Compositae*, whose flowers are of a racemose inflorescence (in *Tanacetum*, moreover, destitute of marginal flowers) and most suitable for this mode of pollen collection. *Fidelia* is oligotropic to *Ficoideae (Aizoaceae)* ([1],[2]), the inflorescence of part of its species also being similar to that in *Compositae*. *Parafidelia* is oligotropic to *Campanula*—similar to the forms *Ficoideae*.

Many other morphological characters are known to occur and develop convergently among the bees, for instance, reduction of the number of radiomedial cells *(Eucera—Tetralonia* and various *Panurgidae*), increase of the free, veinless apical part of the wing *(Anthophora*, some of the *Panurgidae)*, the appearance of outgrowths on the sheath and mandibulae *(Prosopis*, *Osmia)*. Finally, all morphological characters, by which all parasitic bees are arbitrarily grouped together, are an adaptation to their mode of life.

The foregoing is conducive to the statament that the *Fideliidae* are a highly specialized, oligotropic family of bees, closely approaching the family *Colletidae* (s. lato) and having no direct and close affinity with the group of so-called higher bees, particularly *Gastreligidae*. Thus, *Fideliidae* offer an excellent example of independent convergent development of a number of those characters, upon the study of which it is still agreed to base the entire system of classification of *Apoidea*, while the study of the genital armature is usually ignored, though it offers a basis and leading characters for the arrangement of a natural system of *Apoidea*.

Colletidae have a world-wide range of distribution, the family *Diphaglossidae*, like the *Proteaceae*, ranging in Australia, S. Africa and S. America. This virtually determines the geological age of the family, belonging, according to Cockerell and Ireland ([5]), to the primitive bees. The habitat of the highly specialized fam. *Fideliidae* is confined but to the Karroo plateau in S. Africa—the domain of *Proteaceae* and *Ficoideae*. Considering the limited area of distribution and high degree of morphological adaptation, *Fideliidae* are to be recognized as a rather ancient group, though geologically even somewhat younger than family *Diphaglossidae*.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

EVOLUTIONSMORPHOLOGIE

ONTOGENETISCHE GRENZEN DES REGENERATIONSVERMÖGENS DER HINTEREXTREMITÄTEN DER KAULQUAPPEN *RANA TEMPORARIA*

Von L. W. POLEŽAJEW

(Vorgelegt von N. W. Nassonov Mitglied der Akademie, d. 29.I.1939)

Beim studieren der Regenerationsprozesse bei den Anuren muss festgestellt werden, in welchem Stadium das Regenerationsvermögen der Kaulquappen schwindet.

Es wird folgende Stadieneinteilung angenommen:

I Stadium—die Hinterextremitäten der Kaulquappe haben noch keine gegliederten Gelenke zwischen Ober- und Unterschenkel. Der distale Teil des Fusses hat die Form eines Schaufelchens (Ia Stadium), oder eines Schaufelchens mit den Zacken der differenzierenden Zehen (Ib Stadium).

II Stadium—Ober- und Unterschenkel bilden einen stumpfen Winkel, die Anlagen der Zehen sind deutlich zu sehen. Die Extremitäten sind schwach entwickelt (IIa Stadium), oder gut entwickelt und von bedeutender Grösse (IIb Stadium).

III Stadium—Ober- und Unterschenkel sind stark entwickelt und bilden einen spitzen Winkel.

Die Hinterextremitäten der Kaulquappe *Rana temporaria* sind nach einigen Verfassern ([1], [5], [6]) schon im zweiten Stadium regenerationsunfähig, nach anderen ([4], [2], [9])—im III Stadium. Diese Meinungsverschiedenheit hängt, wie weiter erwiesen wird, von der Verschiedenheit der Arbeitsmethoden der genannten Verfasser ab.

Beim studieren der Regenerationsprozesse bei den Anuren habe ich([7]) beobachtet, dass das Resultat der Regeneration von der Amputationshöhe abhängt: bei proximaler Amputation regeneriert die Extremität nicht so gut, als bei distaler. Dieselbe Erscheinung hat Giacomini ([3]) in seiner, der Untersuchung der Metamorphoseprozesse bei den Anuren gewidmeten, Arbeit flüchtig berührt.

Auf das Resultat der Regeneration wirken auch die Unterhaltsbedingungen. In der vorliegenden Arbeit werden die Wirkungen der Amputationshöhe und anderer Faktoren auf die ontogenetische Grenzen des Regenerationsvermögens der Hinterextremitäten bei den Kaulquappen klargelegt.

Als Untersuchungsmaterial dienten die aus einem kalten Bach in der Umgegend der Kropotowschen Biologischen Station (bei Kaschira) geholten Kaulquappen *Rana temporaria*. Der Versuch wurde im Sommer 1938 vorgenommen. Die Temperatur schwankte zwischen 28—37°C. Die Kaulquappen wurden zu je 20—30 Stück in Aquarien von 2—3 Liter Umfang gehalten.

Das Wasser wurde alle 1—2 Tage gewechselt. Die Kaulquappen wurden mit Fadenalgen (*Spirogyra*) gefüttert.

Auf verschiedenen Stadien wurden den Kaulquappen gleichzeitig beide Hinterextremitäten amputiert: die rechte im Proximalteil des Oberschenkels, die linke—im Proximalteil des Unterschenkels (am Kniegelenk). Der Versuch dauerte ungefähr einen Monat. Die Resultate sind in Tabelle zusammengefasst. Als Kriterium galt entweder volle Regeneration der typischen Extremität, oder auch teilweise Regeneration, wenn die regenerierte Extremität einige Defekte aufwies, z. B. das Fehlen von 1—2—3 Zehen.

Amputationshöhe	Volle Regeneration in den Stadien:		Volle und teilweise Regeneration in den Stadien:		
	Ib	IIa	Ib	IIa	IIb
Proximalteil des Oberschenkels	45.4% (38 Fälle)	8.6% (18 Fälle)	53.7% (44 Fälle)	9.6% (20 Fälle)	—
Proximalteil des Unterschenkels . . .	90.7% (76 Fälle)	42.6% (89 Fälle)	109% (82 Fälle)	52.1% (109 Fälle)	4.6% (3 Fälle)
Distalteil des Unterschenkels	—	82.3% (14 Fälle)	—	—	—

Auf gleicher Amputationshöhe—im Proximalteil des Unterschenkels—fand volle Regeneration der Hinterextremitäten in folgenden Stadien statt: Stadium Ib—90.7%; Stadium IIa—42.6%; im Stadium IIb fand nur teilweise Regeneration in 4.6% den vorliegenden Fälle statt, und im III Stadium ging die Regeneration in keinem einzigen der 57 Fälle vor sich.

In ein und demselben Stadium—Stadium IIa—fand volle Regeneration bei Amputation im Proximalteil des Oberschenkels in 8.6% der vorliegenden Fälle statt; bei Amputation im Proximalteil des Unterschenkels—in 42.6% und bei Amputation im Distalteil des Unterschenkels nach Angaben Borssuks—in 82.3% der Gesamtfälle.

Aus den angeführten Angaben sehen wir deutlich, dass das Resultat der Regeneration der Extremitäten bei der Kaulquappe sowohl vom Stadium, in dem die Amputation vorgenommen wurde, wie auch von der Amputationshöhe abhängt.

Es entsteht die Frage, ob die rechte und linke Extremität bei derselben Kaulquappe gleich regenerieren. In Betreff dessen habe ich folgende Angaben erzielt. Den Kaulquappen wurden im Stadium Ib beide Hinterextremitäten gleichzeitig auf Kniegelenkhöhe amputiert. Von 88 Fällen regenerierte kein einziges Füsschen in einem Fall; beide Füsschen regenerierten vollständig in 66 Fällen; beide Füsschen regenerierten teilweise in 11 Fällen; der rechte Fuss regenerierte besser als der linke in 5 Fällen, der linke besser als der rechte auch in 5 Fällen. Folglich, regenerierten im ganzen Versuch beide Füsschen bei gleicher Amputationshöhe ganz gleich. Jedoch kann in einzelnen Fällen (5.6%) das eine Pfötchen bei ein und demselben Tierchen aus unbekannten Gründen besser als das andere regenerieren. Eine ähnliche Erscheinung wurde bei Regenerationen bei verschiedener Amputationshöhe beobachtet. Bei Amputation der rechten Hinterextremitäten im Proximalteil des Oberschenkels, und der linken im Proximalteil des Unterschenkels überwog in 43 von 82 Fällen im Stadium Ib die Regeneration des linken Füsschens diejenige des rechten; aber es traf auch der entgegengesetzte Fall in—in 2 Fällen überwog die Regeneration des rechten Füsschens diejenige des linken. Ein gleicher Fall unter denselben Versuchsbedingungen wurde auch für das Stadium IIa festgestellt: das linke Füsschen

regenerierte in 92 von 209 Fällen besser als das rechte, und das rechte in 3 Fällen besser als das linke.

Ob die Unterhaltsbedingungen der Tiere die Ergebnisse der Regeneration beeinflussen? Die Kaulquappen wurden in zwei Gruppen geteilt: die erste Gruppe wurde unter obengenannten günstigen Bedingungen gut gehalten, die andere—schlecht: das Wasser wurde selten gewechselt, nur alle 5—7 Tage, auf jede Umfangseinheit des Wassers kamen doppelt so viele Kaulquappen, als in der ersten Gruppe. Die Wirkungen der schlechten Unterhaltsbedingungen—Ansammlung von Exkretionsprodukten, Mangel an Sauerstoff, grosse Gedrängtheit—zeigten sich darin, dass die Kaulquappen sich sehr schlecht entwickelten. Sie wurden siech, hörten auf zu essen und zu wachsen, der Bauch fiel ein. Beim Amputieren im Proximalteil des Oberschenkels trat vollständige Regeneration ein, die bei den Kaulquappen der I-ten Gruppe 53.7% ergab, bei den Kaulquappen der II-ten—14.5%.

Ich nehme an, dass die Meinungsverschiedenheit in betreff der ontogenetischen Grenzen des Regenerationsvermögens der Kaulquappe hauptsächlich darauf beruht, dass die Verfasser nicht mit der Bedeutung der Amputationshöhe für die Regeneration rechnen. Z. B., wenn Liosner die ontogenetischen Grenzen des Regenerationsvermögens feststellt, amputiert er die Extremitäten auf Kniegelenkhöhe, Borssuk dagegen—auf Fusshöhe. Selbstverständlich sind die erhaltenen Resultate bei den beiden verschieden. Andere Verfasser stellen das Stadium nicht fest, was auch Anlass zu einigen Missverständnissen gibt. Jakovleva ([9]), z. B., gibt die Abbildung eines typischen II-ten Stadiums, bezeichnet es aber als III-tes Stadium (7-es nach ihrer Nomenklatur).

Unsere Angaben stellen fest, dass die Regeneration bei Extremitätsamputationen im Proximalteil des Unterschenkels im I-ten Stadium gut verläuft; im III-ten Stadium gar nicht vor sich geht; als unbedeutender Prozentsatz, und nur teilweise, im Stadium IIb auftritt, und sich im Stadium IIa in 42—43 Fällen pro hundert zeigt.

Histologische Untersuchungen der Kaulquappenextremitäten haben gezeigt, dass die Distalteile später differenzieren, als die Proximalteile. Während im Stadium IIa das Hüftgelenk schon entwickelt und der Proximalteil des Oberschenkels vollständig verknorpelt war, die Muskeln ihrerseits sich schon zu quergestreiften Fasern ausgebildet hatten, hatte sich im Gebiet des Knies das Gelenk noch nicht gebildet, das Skelett war noch nicht vollständig verknorpelt und die Muskelanlage fing erst an sich zu Fasern zu differenzieren. Wie ich schon früher ([7], [8]) angedeutet habe, kann der Regenerationsprozess der Extremitäten nur bei einem bestimmten Grad histologischer Differenzierung der Extremitäten vor sich gehen. Folglich können die Angaben der vorliegenden Arbeit dadurch motiviert werden, dass, infolge eines höheren Grades von histologischer Differenzierung der Gewebe, die Regeneration auf Proximalhöhe weniger gut verläuft, als auf distaler Höhe.

Das rechte, sowohl wie das linke Paar Extremitäten der Kaulquappen besitzen gleiches Regenerationsvermögen. Aber wie oben erwähnt, in manchen Fällen (5.6%) überwiegt die Regeneration der einen Extremität diejenige der anderen. Dieses Faktum ist schwer zu erklären. Dennoch muss bei praktischer Durchführung von Regenerationsexperimenten an den Extremitäten der Kaulquappen damit gerechnet werden. Dieses Faktum nötigt uns zu doppelter Kontrolle: Kontrolle über den Regenerationsprozess jedes Paares Extremitäten bei jedem einzelnen Individuum, und Kontrolle über den Regenerationsprozess der Extremitäten in der Masse.

Noch ein letzter Punkt muss erwähnt werden, nämlich, dass es unumgänglich notwendig ist beim Vergleichen der Resultate der Regenerationsversuche an den Kaulquappen mit den Unterhaltsbedingungen der Untersuchungstiere während der Versuchszeit zu rechnen.

Schlussfolgerungen

1. Der Regenerationsprozess der Hinterextremitäten der Kaulquappen verläuft auf Distalhöhe besser, als auf Proximalhöhe, was augenscheinlich dadurch bedingt wird, dass die histologische Differenzierung im Proximalteil der Extremität früher eintritt, als im Distalteil.

2. Beim Amputieren im proximalen Teil des Unterschenkels verlieren die Extremitäten der Kaulquappe der *Rana temporaria* ihr Regenerationsvermögen vollständig im Stadium IIa, und teilweise im Stadium IIb.

3. Sowohl die rechte, wie die linke Hinterextremität der Kaulquappe besitzen gleiches Regenerationsvermögen. Aber etwa 6% der Gesamtfälle ergaben, dass die Regeneration der rechten Extremität im Stadium Ib besser verlief, als die der linken, oder auch umgekehrt, die der linken besser, als die der rechten.

4. Die ontogenetischen Grenzen des Regenerationsvermögens werden auch durch die Unterhaltsbedingungen der Kaulquappen während der Untersuchungszeit beeinflusst. Grosse Gedrängtheit, Ansammlung von Exkretionsprodukten und Mangel an Sauerstoff setzen das Regenerationsvermögen herab.

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Eingegangen
d. 3. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

EVOLUTIONARY MORPHOLOGY

ON THE MODE OF RESTORATION OF REGENERATIVE POWER IN THE LIMBS OF TAILLESS AMPHIBIANS

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(Communicated by N. W. Nassonov Member of the Academy, 29.I.1939)

If both hind limbs of *Rana temporaria* tadpoles are, at the stage of their losing regenerative power, amputated simultaneously and on the same level, the stump of the right limb being then traumatized, the right limb regenerates and the left one does not ([4], [5], [6], [9]). The restoration of regenerative power depends on the restoration of the accumulation phase, viz., on the formation of the regenerative blastem on the surface of the amputated limb.

What is the mode of restoration of the regenerative power? In other words, what changes in the stump of the limb amputated determine the formation of the regenerative blastem on the amputation wound surface?

The experimental material has been furnished by *Rana temporaria* tadpoles. The experiment has been carried out in summer 1935 and 1938. The animals have been taken at the stage when thigh and shank form an obtuse angle and are developed faintly (stage IIa) or strongly (stage IIb), and when the limbs lose their regenerative power after an amputation has been practiced at the proximal part of the shank. Both hind limbs would be amputated simultaneously on the same level, in the proximal part of the shank. The stump of the right limb would then be traumatized, the left one serving as control. The results of the experiment have been divided into two groups: the positive (+) when the right limb operated upon regenerated better than the control one, and the negative (—) when the right limb regenerated not so well as the left one.

In my former work I traumatized the stump of the limb amputated by means of piercing it with a needle in longitudinal or transverse direction.

The traumatization brought about a dedifferentiation of the tissues and led to cellular proliferation. Thus, both factors were acting simultaneously. My aim is now to examine the significance of each of these factors taken separately. For this purpose, I applied the method of traumatization of different parts of the stump: the proximal and the distal one; in addition to it, I resorted to the method of counting the mitoses that occurred in different parts of the regenerating limb: thigh, shank stump, tissues of the amputation wound surface and in the regenerative rudiment.

The traumatization of the proximal half of the amputated limb stump was performed by means of piercing it with a needle in transverse direction. In this case, the tissues of the amputation wound surface, being unaffected by the mechanical factor, should undergo no dedifferentiation, they should,

Stage	Traumatization	Operated	Fixed at early stages	Remained to be counted	No limb regenerated	Both limbs regenerated	Both limbs regenerated partly	The right limb regenerated, the left one did not	The left limb regenerated, the right one did not	The right limb regenerated wholly, the left one partly	The left limb regenerated wholly, the right one partly	The right limb regenerated partly, the left one did not regenerate at all	The left limb regenerated partly, the right limb did not regenerate at all	Results of regeneration + Number of cases	Results of regeneration + %	Results of regeneration − Number of cases	Results of regeneration − %	Notes
		1	2	3	4	5	6	7	8	9								
IIa	Distal-proximal	—	17	104	29	36	5	18	—	8	1	6	1	32	30.8	2	1.9	Data of 1933—1938
	Distal	97	6	84	16	16	4	26	—	7	2	13	—	46	54.5	2	2.4	Data of 1938
	»	117	15	57	7	13	7	10	—	10	4	4	2	24	42.1	6	10.5	Data of 1935
	»	214	21	141	23	29	11	36	—	17	6	17	2	70	49.6	8	5.7	Total
	Proximal	97	6	86	50	10	3	3	4	2	2	7	5	12	13.9	11	12.8	Data of 1938
	»	129	13	74	12	18	22	3	3	3	5	6	2	12	16.2	10	13.5	Data of 1935
	»	226	19	160	62	28	25	6	7	5	7	13	7	24	15.0	21	13.1	Total
IIb	Distal	40	—	35	27	—	—	—	—	—	—	7	1	7	20.0	1	2.9	Data of 1938
	Proximal	40	—	30	27	—	1	—	—	1	—	1	—	2	6.7	—	—	Data of 1938

however, be affected by the waste products which form very abundantly in the proximal part of the organ. If the mechanism of the regenerative blastem formation consisted in merely stimulating the cells to proliferate over the amputation wound surface, the limb should regenerate.

When traumatizing the distal part of the amputated limb stump, the piercing was effected in longitudinal direction. In this case, the tissues of the amputation wound surface, besides proliferating under the action of the waste products, should dedifferentiate owing to mechanical injury. If the mechanism of the regenerative blastem formation consisted in a mere accumulation of cellular material, accumulation in the main unaccompanied by proliferation, but attained through its disengaging itself from the tissue connexions directly, the limb should regenerate.

The traumatization of both the proximal and the distal parts of the stump has been performed by means of the same number (15—20) of piercings.

In order to study the early stages of the regenerative process the limbs were fixed at two terms: on the 3rd and the 7th day after amputation. Later on, they were cut up frontally into serial sections and stained with Heidenhain haematoxylin and eosin.

The results of the experiments are summarized in the table (p. 649).

The data 4, 6 and 8 of the table are marked as positive (+) results of the regeneration, the data 5, 7 and 9 of the table—as negative (—) ones. The negative results may be accounted for by the fact that in certain number of cases, after a simple amputation (not accompanied by traumatization), the regeneration of one limb may proceed with more success than the regeneration of the other, the ratio of such cases being the same for the right and the left limb ([12]).

The data summarized in this table show that the traumatization of the proximal part of the stump of the limb amputated does practically not lead to the restoration of regenerative power, the ratio of positive results to negative ones being 15.0 : 13.1. Proximal-distal traumatization brings about the restoration of regenerative power, showing 30.8% positive results of regeneration against 2.4% negative ones. Distal traumatization also brings about the restoration of regenerative power even with better success: 49.6% positive results of regeneration against 5.7% negative ones.

The method of traumatization leads to a complete regeneration of the limb at the stage IIa, and to an incomplete regeneration at the stage IIb (20.0% positive results of regeneration against 2.9% negative ones).

Histological examination gave the following results:

A. Changes in the Control Limbs not Traumatized after Amputation. In the thigh the differentiation of the tissues goes on, but single mitoses are observed: about 10 per 20 serial sections. An insignificant decomposition of the cartilaginous substance of the skeleton and a slight reduction of the differentiation of muscle patterns take place in the shank stump. At the second fixation term the decomposition phenomena disappear, histological differentiation becomes more stable. There are few isolated cells, not included in tissue connexions. In the tissues of the amputation wound surface and in the entire mass of mesodermic tissues there are, at both terms of fixation, under the epithelium a great number of mitoses observed—about 90 per 20 serial sections. The number of mitoses is higher where the tissue differentiation is weakened. The limbs do not regenerate. They regenerate, if tissue dedifferentiation takes place, viz. liquefaction of the cartilaginous skeleton and decomposition of muscle patterns into separate cells. The number of mitoses remains all the time about the same as in the non-regenerating limbs.

B. Changes in the Limbs Traumatized Proximally after Amputation. In the traumatized thigh the skeleton remains intact, while the muscle patterns are decomposed, differentiating and disintegrating into separate cells morphologically undistinguishable from the cells of the regenerative blastem. At both terms of fixation, a great number of mitoses are observed in this traumatized zone, about 80 per 20 serial sections. On the 7th day after amputation the cells which have freed themselves from their tissue connexions gradually begin to orientate themselves in a definite way—the structure of destroyed muscle patterns becomes restored. The tissues of the shank change in the same way as in the control limbs; the originally somewhat weakened differentiation strengthens and progresses; there are about as many mitoses as in the control limbs. The limbs do not regenerate. In the cases when they do so, dedifferentiation of tissues takes place.

C. Changes in Limbs, Distally Traumatized after Amputation. In the tissues of the amputation wound surface and in the inflamed and swollen mesodermic tissues of the shank under the epithelium, there are scattered large seats of haemorrages. The skeleton becomes almost entirely liquefied, the cells freed from the perichondrium and the cartilaginous capsules are perfectly viable and divide successfully. The muscle patterns disintegrate into separate cells. At the second term of fixation, three zones can be distinguished: the proximal—thigh—where the tissues differentiation has made a considerable progress and, with the beginning of ossification, striate muscle bundles and a cartilaginous skeleton have formed; the middle zone—the shank stump—where the tissues have completely dedifferentiated, and a homogeneous dense mass of mesenchymatous cells has appeared, among which there remain thickened parts with signs of cartilage forming and thickening with elements of muscle fibres; and the distal zone—hemi-spherical or conical regenerative rudiment, the subepithelial part of which consists of rather loosely arranged mesenchymatous cells which pass directly into the mesodermic tissue of the shank. The number of mitoses is the same at both terms of fixation, approximating that of mitoses in the control limbs which fail to regenerate. The mitotic coefficient—the relation between the number of mitoses and the number of non-dividing cells, expressed in %,—is low for the regenerative rudiment at the period of cell accumulation. The cell accumulation in the rudiment thus proceeds practically without proliferation.

If the differentiation has persisted in a part of the tissues on the wound surface, the regenerative rudiment arises only in the other part which has dedifferentiated.

Proliferation sets forth in the tissues of the amputation wound surface as well as in the mesodermic tissues of the shank which have been severed from the latter; it is brought about by the amputation of the organ, viz. by the action of the waste products. After amputation, the traumatization of the limb stumps does not activate proliferation: the number of mitoses is, on the average, the same both in the experimental and in the control limbs. The regeneration, be it simple amputation or amputation combined with traumatization, takes place only if the tissues have dedifferentiated. At the earlier stage of the formation of regenerative rudiments, the number of mitoses in it is inconsiderable, not exceeding the number of mitoses in the amputation wound surface of the non-regenerating limb. Accumulation of cells in the rudiment proceeds very rapidly and intensively. We must infer from these facts that, in case of traumatization, the restoration of regenerative power depends, in the main, on the tissue dedifferentiation in the region of the amputation wound surface. Mesodermic tissues of the distal part of the

limb, the skeleton and the muscle patterns do liquefy; in this way a powerful source is formed, from which cells swiftly emerge and, streaming towards the epithelium, form a regenerative blastem on the surface of the amputation wound. Proliferation plays there a secondary part. It will be the chief part at more advanced stages, after the dedifferentiation has ceased and the rudiment is saturated with cellular material, flown over to it from out the amputated organ stump. The same phenomenon may be observed in the process of normal regeneration of tadpole limbs and seems to be true for the regeneration of animals capable of regeneration in general. This theory of formation of the regenerative rudiment in the main without proliferation, but at the expense of the affluence of non-dividing cells, freed from the tissue connexions, has been developed by me in my previous communications ([6], [7], [8], [9], [10], [11], [12], [13], [14]) and set against the sole theory that had existed hitherto, according to which the regenerative rudiment forms only under the influence of proliferation. Recently, my theory has been adopted by Jakovleva ([16]), who has shown that in the case of transplantation there takes place dedifferentiation of tissues in the tadpole limbs.

The loss of regenerative power by tadpole limbs is due to the progress of histological differentiation. The latter can have a two-fold significance: 1) it may inhibit the process of accumulation of the cellular material in the regenerative rudiment, while it detains the cells in the tissue connexions (our data), or while suppressing the process of proliferation [([15],[2]), our data]; 2) or else it may inhibit the organologic differentiation ([8], [11], [3]). At the present time, I am examining the question, whether regenerative power can be totally lost independently of histological differentiation, since the regenerative power is known to be influenced by other factors too, as, for instance, by the humoral medium ([1]).

Conclusions. 1. The restoration of regenerative power in the limbs of *Rana temporaria* is attained by traumatization of the distal, but not of the proximal part of the organ amputated.

2. Traumatization is an essential condition of proliferation and dedifferentiation. Proliferation is equally intense in the traumatized and untraumatized amputated limbs. The proliferation by itself alone cannot lead to the restoration of regenerative power.

3. The loss of regenerative power is determined by the progress of histological differentiation, while the restoration of this power is determined by dedifferentiation of mesodermic tissue.

4. Dedifferentiated mesodermic tissue—skeleton and muscle patterns—form the source, out of which the cellular material passes into the regenerative rudiment in the main without proliferation.

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Received
3. II. 1939.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXII, № 9

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ENTWICKLUNG UND REDUKTION DER WEIBLICHEN GESCHLECHTSORGANE BEIM BANDWURM DES SCHWEINES

Von W. R. WEIZMANN

(Vorgelegt von T. I. Schmalhausen, Mitglied der Akademie, d. 13. I. 1939)

Wir kennen noch sehr wenig die natürliche Reduktion der Gewebe und Organe, sowie die Veränderungen der ausgebildeten Organe im ununterbrochenen individuellen Entwicklungsgeschehen. In unseren Untersuchungen beabsichtigten wir zu klären, worin die Reduktion der weiblichen Geschlechtsorgane beim Schweinebandwurm besteht, und wollten gleichzeitig die Zusammenhänge zwischen Ausbildung und Reduktion dieser Organe verfolgen. Für diese Fragestellung sind Bandwürmer besonders geeignete Objekte dank den verschiedenen Entwicklungsstadien, in denen sich die sie bildenden Proglottiden befinden.

Die weiblichen Geschlechtsorgane werden bereits in sehr jungen Proglottiden in Gestalt von drei Zellkomplexen angelegt; diese Zellkomplexe heben sich scharf von dem sie umgebenden Parenchym ab, dagegen aber unterscheiden sich die Zellen der Eierstockanlage in keiner Beziehung von denen der «Dotterstock»- und Uterusanlagen. Die betreffenden Zellen sind von rundlicher oder ovaler Gestalt und besitzen scharfe Ränder, sowie sehr kleine, runde und pyknotische Kerne (Abb. 1). Ihr Plasma sieht aus wie eine einzige Vakuole, in deren Zentrum der sich intensiv färbende Kern liegt. Die Uterusanlage stellt ein langes, homogenes Gebilde dar, ohne jede Spur eines Hohlraumes.

Im Laufe der Entwicklung dieser Anlagen stellen sich folgende Veränderungen in ihren Zellen ein: die sich allmählich vergrössernden Kerne hören auf pyknotisch zu sein, so dass in ihnen einzelne Chromatinkörner sichtbar werden. Die Vakuolisierung des Plasmas schwindet und das Plasma wird fein granuliert und basophil, wobei die Zellgrenzen allmählich undeutlicher werden. Solche Veränderungen treten zuerst in den Eier- und Dotterstockanlagen auf, während die Zellen der Uterusanlage auf diesem Entwicklungsstadium noch wenig verändert sind. Der nächste Entwicklungsschritt wird gekennzeichnet durch eine starke Volumzunahme der zu Syncytien umgebildeten Anlagen, wobei die Eier- und Dotterstockkerne sich noch vergrössern, allmählich heller werden und kleine Nucleolen aufweisen. In der entsprechenden Uterusanlage dagegen behalten die meisten Kerne ihr früheres Aussehen. Im Laufe der weiteren Entwicklung treten in den Eier- und Dotterstockanlagen Zellen auf, die das Aussehen von jungen Oocyten besitzen. Es sind plasmareiche Zellen mit grossen runden bläschenförmigen Kernen, in denen gut entwickelte Nucleolen zu sehen sind. In der entsprechen-

den Uterusanlage treten keine solchen Zellen auf und die meisten Kerne bleiben klein und pyknotisch. Die inneren Uterusteile lockern sich auf, was durch die Nekrose zahlreicher Zellen bedingt wird; dadurch kommt es zur Bildung einer Höhle in der Uterusanlage.

Die darauffolgenden Veränderungen der Eierstöcke bestehen in der Vermehrung und Vergrösserung der Oocyten. Daneben treten im Plasma der Oocyten «Dotterkörner» auf. Zu gleicher Zeit beginnt im «Dotterstock» die Bildung von «Dotterkugeln», die innerhalb der im Syncytialplasma des bettreffenden Organs entstandenen Lakunen liegen. Diese Lakunen und die in ihnen enthaltenen «Dotterkörner» sind anfangs klein, allmählich aber nimmt ihre Grosse zu. Die «Dotterstockkerne» werden etwas kleiner im Vergleich zum vorangehenden Entwicklungsstadium und die jungen Oocyten verschwinden völlig aus diesem Organ.

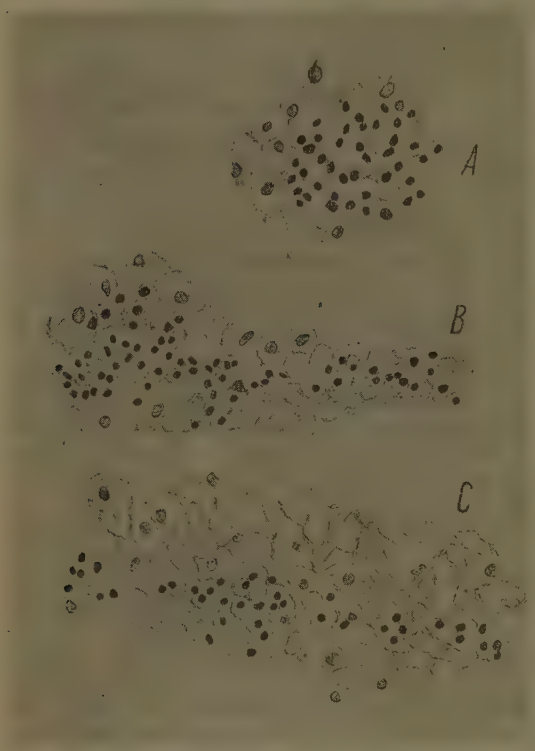

Abb. 1. Querschnitte durch Anlagen des weiblichen Geschlechtssystems einer sehr jungen Proglottis: *A*—Uterus, *B*—Eierstock, *C*—«Dotterstock».

Die entsprechende Uterusanlage bildet ein röhrenförmiges Organ, in dessen Wandungen die meisten Kerne ihre ursprüngliche Beschaffenheit beibehalten haben (Abb. 2, *A*).

Die weitere Entwicklung des Uterus ist gekennzeichnet durch den Beginn der Verästelung; dieser Vorgang wird bedingt durch das Einwachsen der Uteruswandungen in das sie umgebende Parenchym, das an den betreffenden Stellen aufgelockert und degradiert wird. Dadurch kommt es zur Entstehung von Höhlen, die teilweise oder ganz von den Uteruselementen begrenzt sind. Bei der Bildung im Uterus der zentralen und seitlichen Höhlen gelangen in diese Zellen oder Zellkomplexe, die sich von den Uteruswänden losgelöst haben. Die Kerne der eliminierten Zellen sind etwas vergrössert, rund und lassen scharf abgegrenzte kleine Nucleolen und einzelne Chromatinkörner erkennen. Die Elimination der Zellen beginnt noch vor der Entstehung der Seitenzweige und zu dem Zeitpunkt, wenn die Bildung der letzteren einsetzt, ist die gesamte zentrale Höhle des unverästelten Uterus mit zahlreichen Zellen gefüllt, die sich in ihrer Struktur und besonders in ihrer Grösse stark von den grossen Oocyten unterscheiden, wie sie im benachbarten Eierstock, niemals aber in der Uterushöhle zu finden sind.

Zu dem Zeitpunkt, wenn die Verästelung des Uterus beginnt, kommt es in den Eierstockkernen zu einer starken Vergrösserung der «Dotterkörner»; gleichzeitig setzt in den Oocyten eine allmähliche Karyolysis ein, und die

Oocyten werden zu kernlosen Gebilden, die grosse Dottereinschlüsse enthalten. Das Ergebnis dieser Prozesse besteht darin, dass grosse Eierstockbezirke zu Haufen von «Dotterkörpern» umgebildet werden (Abb. 2,*B*, *C*). Im entsprechenden «Dotterstock» beobachtet man wieder eine Abnahme der Grösse und Zahl der Kerne, sowie eine Vergrösserung der Lakunen und der in ihnen enthaltenen «Dotterkörner».

Die meisten Zellen, die die Höhle des sich verästelnden Uterus füllen, werden grösser und nehmen die verschiedenste Gestalt an. Dabei werden in einigen Zellen die ebenfalls vergrösserten Kerne bläschenförmig, die anderen dagegen behalten ihre frühere Struktur. Im Plasma beider Zellarten entstehen manchmal recht grosse «Dotterkörper». Die grosskörnigen Zellen ähneln den Oocyten der Eierstöcke und weisen dieselben Veränderungen auf, d. h. bilden sich zu «Dotterkörpern» um oder degradieren unmittelbar. Die Kerne der kleinkernigen Zellen dagegen sehen vollkommen lebensfähig aus. Dasselbe lässt sich von den kleinen Kernen der grossen Zellen sagen, die «Dottereinschlüsse» aufweisen. Die «Dotterkörper» entstehen öfters unmittelbar innerhalb der syncytialen Uteruswand, wo sie zwischen kleinen dunklen Kernen eingelagert sind. Somit entsteht der «Dotter» nicht nur auf Kosten der Eier und «Dotterstockelemente», sondern auch auf Kosten der Uterusbestandteile.

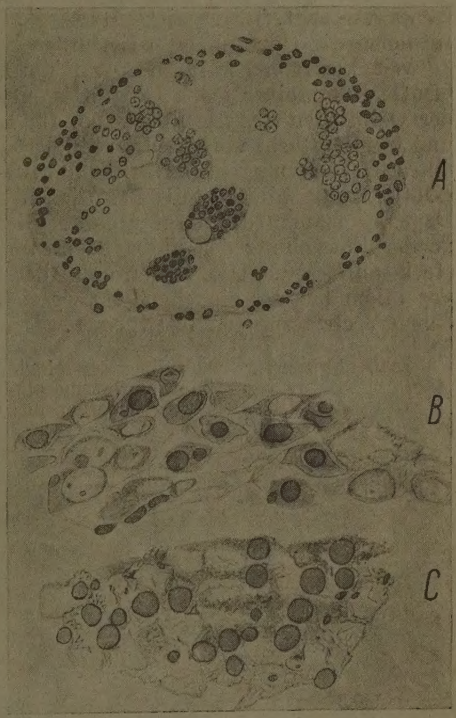

Abb. 2.*A*—Querschnitt durch den Uterus vor dem Beginn der Verästelung: *C*—Eierstockteile der Proglottis, dessen Uterus zu ästeln began; *B*—der dem Ootype direkt anliegende Teil.

Alle beschriebenen Vorgänge führen zu folgendem Ergebnis. Die Eierstöcke werden zu Ansammlungen von «Dotterkugeln», dasselbe Schicksal erfahren auch die «Dotterstöcke», deren Kerne sekundär pyknotisch werden und allmählich verschwinden.

Die dünner werdenden Uteruswände bestehen aus einer Schicht Zellen mit länglichen, sehr kleinen und pyknotischen Kernen. Der Uterusinhalt besteht aus degradierten grossen bläschenförmigen Kernen, nekrotischem Plasma, zahlreichen «Dotterkörpern» und den oben beschriebenen kleinen Zellen. Alle diese Gebilde sind völlig regellos untereinander vermischt und es lassen sich darunter keine Gebilde erkennen, die man als reife, in Furchung begriffene Eier auffassen könnte.

In der weiteren Entwicklung der Proglottiden verschwinden allmählich die Eier- und «Dotterstockreste» vollständig und werden durch Parenchym ersetzt. Die Reste des «Dotterstocks» bleiben länger erhalten, als die des Eierstocks. Was den Uterus betrifft, so werden die in ihm enthaltenen kleinen Zellen mit runden Kernen in Gruppen zu zwei, drei oder vier Zellen

verteilt, wobei aber gleichzeitig noch reichlich Einzelzellen übrig bleiben. Die Endstadien der Entwicklung werden dadurch gekennzeichnet, dass die «Dottermenge» im Uterus allmählich ab-, die Zahl der vielgliedrigen Zellgruppen dagegen zunimmt. Mit anderen Worten: der allmähliche Schwund des «Dotters» verläuft paralell zur Teilung der eliminierten kleinen Uteruszellen, die als Keimzellen bezeichnet werden können. Mit der Teilung (Furchung) der Keimzellen treten die sich entwickelnden und abrundenden Embryonen immer schärfer auf dem sie umgebenden Hintergrund hervor. Der rundliche Embryo bildet an seiner Oberfläche eine Schale, die sich später verdickt. Schliesslich entstehen in der Uterushöhle zahlreiche Kokons, in denen die Oncosphaeren liegen.

Wir kommen somit zu folgenden Ergebnissen: Uterus, Eierstock und «Dotterstock» einer jeden Proglottis entwickeln sich aus homogenen Anlagen der weiblichen Geschlechtsorgane. Im Laufe der Entwicklung werden Eier- und «Dotterstock» vollständig reduziert und zu Haufen in Resorption begriffenen Dotters umgebildet. Die Uteruszellen werden teilweise ebenfalls in «Dotter» umgebildet, teilweise dienen sie aber zu Fortpflanzungszwecken. Dagegen nehmen die ausgebildeten Eier- und «Dotterstöcke» keinen Anteil an der Entstehung der Embryonen. Wir müssen also den Uterus als Kernstock auffassen. Die Bedeutung der beschriebenen Verhältnisse erhellt vor allem bei der eingehenden vergleichenden Untersuchung von verschiedenen Vertretern der Plattwürmer.

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Institut für Evolutionsmorphologie.
Akademie der Wissenschaften der UdSSR.

Eingegangen
d. 26. I. 1939.

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